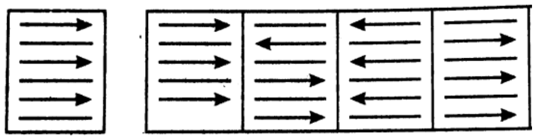
INSTRUCTIONS

1. You have opted for English as medium of question paper. This test Booklet contains one hundred and twenty (20 Part' A' +40 Part 'B' +60 Part 'C') Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 25 and 20 questions from part 'A' 'B' and 'C' respectively. If more than required number of questions are answered, only first 15, 25 and 20 questions in Parts 'A' 'B' and 'C' respectively, will be taken up for evaluation.
2. Each question in Part 'A' carries 2 marks, Part 'B' 3 marks and Part 'C' 4.75 marks respectively. There will be negative marking @0.5 marks in Part 'A' and @ 0.75 marks in Part 'B' for each wrong answer and no negative marking for Part 'C'.
3. Below each question in Part 'A' and 'B', four alternatives or responses are given. Only one of these alternatives is the "correct" option to the question. You have to find, for each question, the correct or the best answer. In Part 'C' each question may have "**ONE**' or '**MORE**' correct options in Part 'C'. Credit in a question shall be given only on idenfication of 'ALL' the correct options in Part 'C'. No credit shall be allowed in a question if any incorrect option is marked as correct answer.

Part - A

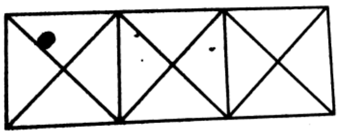
1. Choose the correct mirror image of the figure (A) from amongst the four alternatives.



(A) (1) (2) (3) (4)

2. Choose the correct water image out of given four alternatives. 96FSH52

3. How many triangles are there in the given figure ?



1. 28 2. 24
3. 25 4. 26

4. Pointing to a lady in the photograph, Shaloo said, "Her son's father is the son-in-law of my mother". How is Shaloo related to the lady ?
1. Aunt 2. Sister
3. Mother 4. Cousin

5. I was born on August 11. Mohan is younger to me by 11 days. This year's Independence Day falls on Monday. The

day on which Mohan's birthday will fall this year will be

1. Monday 2. Tuesday
3. Sunday 4. Thursday

6. At what time between 7 and 8 o'clock, will the hands of a clock be in the same straight line but, not together ?

1. 5 min past 7 2. $5\frac{2}{11}$ min past 7
3. $5\frac{3}{11}$ min past 7 4. $5\frac{5}{11}$ min past 7

7. Facing towards South, Ram started walking and turned left after walking 30 m, he walked 25 m and turned left and walked 30 m. How far is he from his starting place and in which direction ?

1. At the starting point only
2. 25 m, West
3. 25 m, East
4. 30 m, East

8. A man is standing on the desk of a ship, which is 8m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and height of the hill.

1. 8m, 32m 2. $8\sqrt{3}$ m, 16m
3. $8\sqrt{3}$ m, 24m 4. $8\sqrt{3}$ m, 32m

9. The height of a cone is 30cm. A small cone is cut off at the top by a plane parallel to its base. If its volume is $\frac{1}{27}$ of the volume of the given cone, at what height above the base the section is made.

1. 10 cm 2. 20 cm
3. 30 cm 4. 40 cm

10. A sum was put at simple interest at a certain rate for 3 years. Had it been put at 2% higher rate, it would have fetched Rs. 360 more. Find the sum.

1. Rs. 7,000 2. Rs. 8,000
3. Rs. 6,000 4. Rs. 10,000

11. Present ages of Sameer and Anand are in the ratio of 5 : 4 respectively. Three years hence, the ratio of their ages will become 11 : 9 respectively. What is Anand's present age in years ?

1. 24 2. 27
3. 40 4. Cannot be determined

12. The last significant bit of an 8 bit binary number is zero. A binary number whose value is 8 times the previous number was :

1. 12 bits ending with three Zeros.
2. 11 bits ending with four Zeros.
3. 11 bits ending with three zeros.
4. 12 bits ending with four Zeros.

13. Nisha and Jaya both distribute Rs. 100 respectively in charity. Nisha distributes money to 5 more people than Jaya and Jaya gives each Rs. 1 more than Nisha. How many people are recipients of the Charity ?

1. 45 2. 60
3. 90 4. None of these

14. A batsman in his 12^{th} innings, makes a score of 63 runs and thereby increases his average score by 2. The average of his score after 12^{th} innings is :

1. 41 2. 42
3. 34 4. 35

15. Find the missing letter :

C	F	I	L	O
A	C	E	G	I
D	E	F	G	H
Z	X	V	T	R
W	T	Q	N	?

1. J 2. K
3. L 4. M

16. What is next term in the sequence

$4, 6, 9, 13\frac{1}{2}$?

1. $17\frac{1}{2}$ 2. 19
3. $20\frac{1}{4}$ 4. $22\frac{3}{4}$

17. A is 50% as efficient as B. C does half of the work done by A and B together. If C alone does the work in 40 days, then, A, B and C together can do the work in ?

1. $13\frac{1}{3}$ days 2. 15 days
3. 20 days 4. 30 days

18. November 9, 1994 was a Wednesday. Then which of the following is true ?
1. November 9, 1965 is a Wednesday and November 9, 1970 is a Wednesday.
2. November 9, 1965 is not a Wednesday and November 9, 1970 is a Wednesday.
3. November 9, 1965 is a Wednesday and November 9, 1970 is not a Wednesday.
4. November 9, 1965 is not a Wednesday and November 9, 1970 is not a Wednesday.

19. A $12\text{m}\times 4\text{m}$ rectangular roof resting on four 4m tall thin poles. Sunlight falls on the roof at an angle of 45° from the east, creating a shadow on the ground. What will be the area of the shadow ?

1. 24m^2 2. 36m^2
3. 48m^2 4. 60m^2

20. Starting from a point A you fly one mile south, then one mile East, then one mile north which brings you back to point A. Point A is not the north pole. Which of the following must be true ?
1. You are in the Northen Hemisphere.
2. You are in the Eastern Hemisphere.
3. You are in the Western Hemisphere.
4. You are in the Souther Hemisphere.

Part - B

Unit – I

21. Let A be a $n\times n$ non singular real symmetric matrix then
1. A must be congruent to A^{-1}.
2. A must be similar to A^{-1}.
3. Both are true.
4. Both are false.

22. The rank of the matrix $\begin{bmatrix} a & p & b & q \\ 0 & a & 0 & b \\ c & r & d & s \\ 0 & c & 0 & d \end{bmatrix}$ is 3 if

1. $ad = bc$ 2. $ac = bd$
3. $ab = cd$ 4. None of these

23. Let A, B, C be three matrices such that BA, AC and BAC exist and A and BA are of same rank then

1. BA and AC are of same rank.
2. AC and BAC are of same rank.
3. Both are true.
4. Both are false.

24. Let A be a 10×10 real matrix then which of the following is true :

1. $\rho\left(A^8\right)=\rho\left(A^9\right)$
2. $\rho\left(A^9\right)=\rho\left(A^{10}\right)$
3. $\rho\left(A^{10}\right)=\rho\left(A^{11}\right)$
4. $\rho\left(A^8\right)=\rho\left(A^7\right)$

25. If $f(x)=(x-a)(x-b)(x-c)(x-d)$ then

$$\begin{vmatrix} a & x & x & x \\ x & b & x & x \\ x & x & c & x \\ x & x & x & d \end{vmatrix} \text{ equals}$$

1. $f(x)+f'(x)$ 2. $f(x)-f'(x)$
3. $f(x)+x f'(x)$ 4. $f(x)-x f'(x)$

26. The value of the determinant

$$\begin{vmatrix} 0 & x & y & z \\ -x & 0 & r & q \\ -y & -r & 0 & p \\ -z & -q & -p & 0 \end{vmatrix} \text{ is}$$

1. $(px+qy+rz)^2$
2. $(px+qy-rz)^2$
3. $(px-qy+rz)^2$
4. $(px-qy-rz)^2$

27. Let $I=[0,1]\cup[2,3]\subset\mathbb{R}$. $x\in\mathbb{R}$, let $\varphi(x)=\text{dist}(x,I)=\inf\{|x-y|: y\in I\}$. Then

1. $\varphi(x)$ is discontinuous somewhere on $\mathbb{R}$.
2. $\varphi(x)$ is continuous on $\mathbb{R}$ but not differentiable exactly at $x=0$.
3. $\varphi(x)$ is continuous on $\mathbb{R}$ but not differentiable exactly at finitely many points.
4. $\varphi(x)$ is differentiable on $\mathbb{R}$.

28. Which of the following is best

1. $e^\pi > \pi^e$ 2. $e^\pi < \pi^e$
3. $e^\pi \geq \pi^e$ 4. $e^\pi \leq \pi^e$

29. Let $f:[0,\infty)\to\mathbb{R}$ be a differentiable function such that $f(0)=0$ and f' is increasing then

1. $\frac{f(x)}{x}$ is decreasing
2. $\frac{f(x)}{x}$ is increasing
3. $\frac{f(x)}{x}$ is not monotonic
4. $f(x)>0$ for all $x\in[0,\infty)$

30. The interval of convergence of the series $1-\frac{1}{2}x+\frac{1\cdot3}{2\cdot4}x^2-\frac{1\cdot3\cdot5}{2\cdot4\cdot6}x^3+....$ is

1. $(-1,1)$ 2. $(-1,1]$
3. $[-1,1)$ 4. $[-1,1]$

31. Let A be a open subset of $\mathbb{R}$, $A\neq\phi$, $A\neq\mathbb{R}$. Then A is

1. the closure of the interior of A.

2. a countable set.
3. a compact set.
4. an uncountable set.

32. A function $f:\mathbb{R}^2 \to \mathbb{R}$ is defined by $f(x,y) = xy^2$. Let $v = (1,2)$ and $a = (a_1, a_2)$ be two elements of $\mathbb{R}^2$. The directional derivative of f in the direction of v at a is :

1. $a_1 + 2a_2$
2. $a_2 + 2a_1$
3. $a_1 + a_2$
4. None

Unit - II

33. The integral $\int_C \frac{z+2}{z} dz$, where C denotes the semi circle $z = 2e^{i\theta}, \pi \le \theta \le 2\pi$ equals

1. $-4+2\pi i$
2. $4+2\pi i$
3. $4-2\pi i$
4. $-4-2\pi i$

34. The image of the semi infinite strip $x > 0,\ 0 < y < 1$ under the mapping $f(z) = \frac{i}{z}$ is

1. $\left(u - \frac{1}{2}\right)^2 + v^2 > \frac{1}{4},\ \ u > 0,\ \ v > 0$
2. $\left(u - \frac{1}{2}\right)^2 + v^2 < \frac{1}{4},\ \ u > 0,\ \ v > 0$
3. $\left(u - \frac{1}{2}\right)^2 + v^2 > \frac{1}{4},\ \ u > 0,\ \ v < 0$
4. $\left(u - \frac{1}{2}\right)^2 + v^2 < \frac{1}{4},\ \ u < 0,\ \ v > 0$

35. Let $f(z) = u(x,y) + iv(x,y)$ is analytic in a domain D then which of the following must be analytic in the domain D

1. $i\ f(z)$
2. $-i\ f(z)$
3. Both
4. None

36. Which of the following is true :

1. $\operatorname{Res}_{z=\pi i} \frac{e^{kz}}{\sinh z} + \operatorname{Res}_{z=-\pi i} \frac{e^{kz}}{\sinh z} = 2\sin k\pi$
2. $\operatorname{Res}_{z=\pi i} \frac{e^{kz}}{\sinh z} + \operatorname{Res}_{z=-\pi i} \frac{e^{kz}}{\sinh z} = -2\sin k\pi$
3. $\operatorname{Res}_{z=\pi i} \frac{e^{kz}}{\sinh z} + \operatorname{Res}_{z=-\pi i} \frac{e^{kz}}{\sinh z} = 2\cos k\pi$
4. $\operatorname{Res}_{z=\pi i} \frac{e^{kz}}{\sinh z} + \operatorname{Res}_{z=-\pi i} \frac{e^{kz}}{\sinh z} = -2\cos k\pi$

37. Which of the following intervals contains a positive integer n which satisfies $n \equiv 3 \pmod{13}$, $n \equiv 7 \pmod{11}$, $n \equiv 5 \pmod{7}$

1. $[600, 800]$
2. $[800, 1000]$
3. $[1000, 1200]$
4. $[1200, 1400]$

38. Which of the following polynomials are irreducible over $\mathbb{Z}_3$

1. $x^5 + x^2 + 2$
2. $x^5 + x^4 + x^3 + x^2 + x + 2$
3. Both of these
4. None of these

39. Let p be a prime number. The order of a p – Sylow subgroup of the group $GL_{100}(\mathbb{F}_p)$ of

invertible 100×100 matrices with entries from the finite field F_p, equals :

1. p^{100}
2. p^{1000}
3. p^{4950}
4. p^{4900}

40. Let $A \subseteq \mathbb{R}^2$ and $X = \mathbb{R}^2 \setminus A$ be subsets with subspace topology inherited from the usual topology on $\mathbb{R}^2$. Then
    1. A is countable dense implies that X is totally disconnected.
    2. A is unbounded implies that X is compact.
    3. A is open implies that X is compact.
    4. A is countable implies that X is path connected.

Unit - III

41. Initial value problem
    $\frac{dy}{dx} = \frac{e^{y^2} - 1}{1 - x^2 y^2}$, $y(-2) = 1$ in interval $[-2.02, -1.98]$ have
    1. Unique solution
    2. Infinite solution
    3. No solution
    4. None

42. Solution of the O.D.E.
    $(D^2 - 2D + 1)y = x \sin x e^x$ is
    1. $y = (A + Bx)e^x + e^x(x \sin x + 2\cos x)$
    2. $y = (A + Bx)e^x - e^x(x \sin x + 2\cos x)$
    3. $y = (A + Bx)e^x - e^x(2\sin x + x\cos x)$
    4. $y = (A + Bx)e^x + e^x(2\sin x + x\cos x)$

43. A general solution of the second order equation $u_{xx} - 4u_{yy} = 0$ is of the form $u(x, y) =$
    1. $f(x) + g(y)$
    2. $f(x + 4y) + g(x - 4y)$
    3. $f(y + 2x) + g(y - 2x)$
    4. None

44. Value of P.I. at $(\pi, 0)$ of the PDE
    $(2D^2 - 5DD' + 2D'^2)z = 5\sin(2x + y)$ is
    1. $\frac{-5\pi}{3}$
    2. $\frac{5\pi}{3}$
    3. 0
    4. None

45. Extremal $y = y(x)$ for the variational problem $J = \int_0^1 (1 + y''^2)dx$ satisfies the O.D.E. is
    1. homogeneous linear differential equation of fourth order
    2. non-homogeneous linear differential equation of fourth order
    3. homogeneous non-linear differential equation of fourth order
    4. homogeneous linear differential equation of order more than fourth order

46. Extremal of the functional
    $\int_0^{\frac{\pi}{2}} (y''^2 - y^2 + x^2)dx$ are
    1. one parameter family of curves
    2. two parameter family of curves
    3. three parameter family of curves
    4. four parameter family of curves

47. Suppose $u \in C^2\left(\overline{B}\right), B$ is the unit ball in $\mathbb{R}^2$, satisfies $\Delta u = f$ in B

$$\alpha u + \frac{\partial u}{\partial n} = g \text{ on } \partial B, \quad g\alpha > 0,$$

Where *n* is the unit outward normal to *B*. If a solution exists then
1. it is unique
2. there are exactly two solutions
3. there are exactly three solutions
4. there are infinitely many solutions

48. The magnitude of the truncation error for the scheme
$f'(x) = Af(x) + Bf(x+h) + Cf(x+2h)$ is equal to
1. $h^2 f'''(\xi)$ if $A = -\frac{5}{6h}, B = \frac{3}{2h}, C = -\frac{2}{3h}$
2. $h^2 f'''(\xi)$ if $A = \frac{5}{6h}, B = \frac{3}{2h}, C = \frac{2}{3h}$
3. $h^2 f'''(x)$ if $A = -\frac{5}{6h}, B = \frac{3}{2h}, C = -\frac{2}{3h}$
4. $h^2 f'''(x)$ if $A = \frac{5}{6h}, B = \frac{3}{2h}, C = \frac{2}{3h}$

Unit - IV

49. A box contains 40 numbered red balls and 60 numbered black balls. From the box, balls are drawn one by one at random without replacement till all the balls are drawn. The probability that the last ball drawn is black equals
1. $\frac{1}{100}$
2. $\frac{1}{60}$
3. $\frac{3}{5}$
4. $\frac{2}{3}$

50. $X_1, X_2, ...$ are independent identically distributed random variables having common density *f*. Assume $f(x) = f(-x)$ for all $x \in \mathbb{R}$. Which of the following statements is correct ?
1. $\frac{1}{n}\left(X_1 + ... + X_n\right) \to 0$ in probability
2. $\frac{1}{n}\left(X_1 + ... + X_n\right) \to 0$ almost surely
3. $P\left(\frac{1}{\sqrt{n}}\left(X_1 + ... + X_n\right) < 0\right) \to \frac{1}{2}$
4. $\sum_{i=1}^{n} X_i$ has the same distribution as $\sum_{i=1}^{n} (-1)^{i} X_i$

51. Let N_t denote the number of accidents up to time *t*. Assume that $\{N_t\}$ is a Poisson process with intensity 2. Given that there are exactly 5 accidents during the time period $[20, 30]$, what is the conditional probability that there is exactly one accident during the time period $[15, 25]$?
1. $\frac{15}{32}e^{-10}$
2. $20e^{-20}$
3. $\frac{10^5}{5!}e^{-30}$
4. $\frac{1}{5}$

52. *X* and *Y* are independent random variables each having the density

$f(t)=\frac{1}{\pi}\frac{1}{1+t^2},\ -\infty<t<\infty$. Then the density function of $\frac{X+Y}{3}$ for $-\infty<t<\infty$ is given by

1. $\frac{6}{\pi}\frac{1}{4+9t^2}$
2. $\frac{6}{\pi}\frac{1}{9+4t^2}$
3. $\frac{3}{\pi}\frac{1}{1+9t^2}$
4. $\frac{3}{\pi}\frac{1}{9+t^2}$

53. Suppose $\{X_1,.....,X_n\},\ n\geq 2,$ is a random sample from the distribution with probability density function

$$f(x;\theta)=\begin{cases}\frac{\theta^\theta}{r(\theta)}x^{\theta-1}e^{-x\theta} & ;\quad x>0\\ 0 & ;\quad x\leq 0\end{cases}$$ with

$\theta>0$. Then the method of moments estimator of θ

1. does not exist
2. is $\frac{n}{\sum_{i=1}^{n}(X_i-1)^2}$
3. is $\frac{n}{\sum_{i=1}^{n}(X_i-\overline{X})^2}$
4. is $\frac{n-1}{\sum_{i=1}^{n}(X_i-1)^2}$

54. Let $X_1,X_2,....,X_n$ for $n\geq 5$ be a random sample from the distribution with probability density function

$$f(x;\theta)=\begin{cases}e^{-(x-\theta)} & \text{if } x>\theta\\ 0 & \text{otherwise}\end{cases}$$

for $\theta>0$. The confidence coefficient of the confidence interval

$$\left[\min\{X_1,...,X_n\}-\frac{\ln 4}{n},\min\{X_1,...,X_n\}+\frac{\ln 2}{n}\right]$$

1. 0.5
2. 0.75
3. 0.95
4. $1-\frac{1}{2^n}$

55. Let X be a random sample from an exponential distribution with mean $\frac{1}{\lambda}$. If λ has a prior distribution with probability density function $g(\lambda)=\begin{cases}\lambda e^{-\lambda} & ;\quad \lambda>0\\ 0 & ;\quad \lambda\leq 0\end{cases}$

Then the Bayes estimator of $\frac{1}{\lambda}$ with respect to the square error loss function is

1. $\frac{2}{X+1}$
2. $\frac{1}{X}$
3. X
4. $\frac{X+1}{2}$

56. Consider the linear statistical model

$y_{ij}=\mu+\tau_i+\varepsilon_{ij};\ i=1,2,....,a;\ \ j=1,2,...,n$

Where μ is unknown, τ_i are independently and identically distributed as $N(0,\sigma_\tau^2),\varepsilon_{ij}$ are independently and identically distributed as $N(0,\sigma^2)$; τ_i and ε_{ij} are independent for all i and j. Note that τ_i is the i^{th} treatment effect. Suppose $SS_{total},SS_{treatment},SS_{error}$ are total sum of squares, total treatment sum of squares and error sum of squares, respectively.

To test : $H_0:\sigma_\tau^2=0$ vs $H_A:\sigma_\tau^2>0$ which of the following statements is not true ?

1. The sum of squares identity is
$SS_{total}=SS_{treatment}+SS_{error}$

2. $SS_{error} \sim \sigma^2 x^2 n(a-1)$

3. Under $H_0, \dfrac{\frac{SS_{treatment}}{a-1}}{\frac{SS_{error}}{n(a-1)}} \sim F_{a-1}, n(a-1)$

4. $E(SS_{error}) = n(a-1)\left(\sigma^2 + n\sigma_\tau^2\right)$

57. Suppose (X_1, X_2) follows a bivariate normal distribution with $E(X_1) = E(X_2) = 0,\ V(X_1) = V(X_2) = 2$ and $Cov(X_1, X_2) = -1$. If

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2}\, dy,$$ then

$P[X_1 - X_2 > 6]$ is equal to

1. $\phi(-1)$
2. $\phi(-3)$
3. $\phi(\sqrt{6})$
4. $\phi(-\sqrt{6})$

58. Consider the problem of drawing a sample of size 2 from a finite population of size 20. the sampling is done with replacement using probability proportional to size sampling scheme. The normal size measures $p_1, ..., p_{20}$ are given by $p_i = \frac{1}{40}$, $i = 1,....,10,\ p_i = \frac{3}{40},\ i = 11,....,20.$

The expected number of distinct units drawn is

1. $\frac{83}{80}$
2. $\frac{157}{80}$
3. $\frac{17}{16}$
4. $\frac{31}{16}$

59. If we interchange two columns of a Latin square design (LSD), then the new design is

1. an LSD
2. a completely randomized design (CRD) but not an LSD
3. a randomized block design (RBD) but not an LSD
4. a balanced incomplete block design (BIBD) but not an LSD

60. Consider the LPP :

Minimize $c^t x$ subject to $Ax = b,\ x \geq 0,$

where $A = \begin{bmatrix} 1 & 1 & 3 & 1 & 2 \\ 0 & -1 & -2 & -3 & 1 \end{bmatrix}$,

$b = \begin{bmatrix} 2 \\ -1 \end{bmatrix},\ c = (2, -1, 1, -9, 0)^t$, and

$x = (x_1, x_2, x_3, x_4, x_5)^t$. Using the revised simplex method with current basis as

$\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$, which of the following statements is correct ?

1. The next entering variable is x_5
2. The solution corresponding to the current basis is optimal
3. The next entering variable is x_4
4. The next entering variable is x_3

Part - C

Unit - I

61. Let $f:[0,1]\to\mathbb{R}$ be a function defined by $f(x)=\sqrt{x}$ then which of the following is not true :

1. f is uniformly continuous on $[0,1]$
2. f is Lipschitz function on $[0,1]$
3. f is continuous on $[0,1]$
4. f is differentiable on $[0,1]$

62. Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function such that $1\le f'(x)\le 2$ for all $x\in\mathbb{R}$ and $f(0)=0$ then which of the following is true :

1. $f(x)\ge x$ for all $x\ge 0$
2. $f(x)\le 2x$ for all $x\ge 0$
3. $f(x)\ge x^2$ for all $x\ge 0$
4. $f(x)\le 2x^2$ for all $x\ge 0$

63. Let V be a finite dimensional real vector space and $T:V\to V$ is a nilpotent linear transformation. If $S=\alpha_0 I+\alpha_1 T+\alpha_2 T^2+....+\alpha_k T^k$ then S is non-singular if

1. $\alpha_0=1$
2. $\alpha_0=-1$
3. $\alpha_0=2$
4. $\alpha_0=-2$

64. Consider non-zero vector spaces V_1,V_2,V_3,V_4 and linear transformations $T_1:V_1\to V_2$, $T_2:V_2\to V_3$, $T_3:V_3\to V_4$ such that $\text{Ker}(T_1)=\{0\}$, $\text{Range}(T_1)=\text{Ker}(T_2)$, $\text{Range}(T_2)=\text{Ker}(T_3)$, $\text{Range}(T_3)=V_4$.

Then which of the following is not true :

1. $\dim V_1+\dim V_2+\dim V_3+\dim V_4=0$
2. $\dim V_1-\dim V_2+\dim V_3-\dim V_4=0$
3. $\dim V_1+\dim V_2-\dim V_3-\dim V_4=0$
4. $\dim V_1-\dim V_2-\dim V_3-\dim V_4=0$

65. For $n\ge 1$, let $g_n(x)=\sin^2\left(x+\frac{1}{n}\right)$, $x\in[0,\infty)$ and $f_n(x)=\int_0^x g_n(t)\,dt$. Then

1. $\{f_n\}$ converges pointwise to a function f on $[0,\infty)$, but does not converge uniformly on $[0,\infty)$.
2. $\{f_n\}$ does not converge pointwise to any function on $[0,\infty)$
3. $\{f_n\}$ converges uniformly on $[0,1]$
4. $\{f_n\}$ converges uniformly on $[0,\infty)$

66. Let A and B are two $n\times n$ real idempotent matrices where $n\ge 2$ then which of the following is true :

1. If $\rho(A)=\rho(B)$ then A and B are similar
2. If $\rho(A)\ne\rho(B)$ then A and B are not similar.
3. If A and B are similar then $\rho(A)=\rho(B)$
4. If A and B are not similar then $\rho(A)\ne\rho(B)$

67. Let $F:\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}$ be the function $F(x,y)=\langle Ax,y\rangle$ where $\langle,\rangle$ is the standard inner product of $\mathbb{R}^n$ and A is a $n\times n$ real

matrix. Here D denotes the total derivative. Which of the following statements are correct ?

1. $\left(DF\left(x,y\right)\right)\left(u,v\right)=\langle Au,y\rangle+\langle Ax,v\rangle$
2. $\left(DF\left(x,y\right)\right)\left(0,0\right)=0$
3. $DF\left(x,y\right)$ may not exist for some $\left(x,y\right)\in\mathbb{R}^n\times\mathbb{R}^n$
4. $DF\left(x,y\right)$ does not exist at $\left(x,y\right)=\left(0,0\right)$

68. If A be a $n\times n$ matrix then which of the following is possible for some A :

1. rank (A) + nullity (A) = n
2. rank (A) – nullity (A) = n
3. nullity (A) – rank (A) = n
4. rank (A) + nullity (A) = $2n$

69. Let $q_1=x^2+y^2+z^2+xy$, $q_2=x^2+y^2+z^2+yz$ and $q_3=x^2+y^2+z^2+zx$ and suppose R_1, R_2 and R_3 denotes their ranges respectively. Then which is/are true :

1. $R_1\subseteq R_2$
2. $R_2\subseteq R_3$
3. $R_3\subseteq R_1$
4. $R_1=R_2=R_3$

70. Let A be a 3×3 real matrix such that $A^3-A^2+A-I=0,$ then which is/are true :

1. Eigen values of A can not be distinct.
2. A may have two distinct eigen values.
3. A may have all eigen values same.
4. A must be non singular.

71. Let a be a positive real number. Which of the following integrals are convergent ?

1. $\int_0^a \frac{1}{x^4}dx$
2. $\int_0^a \frac{1}{\sqrt{x}}dx$
3. $\int_4^\infty \frac{1}{x\log_e x}dx$
4. $\int_5^\infty \frac{1}{x\left(\log_e x\right)^2}dx$

72. Let A and B be $n\times n$ real matrices such that $AB=BA=0$ and $A+B$ is invertible, which of the following are always true ?

1. $\text{rank}\left(A\right)\le n-\text{rank}\left(B\right)$
2. $\text{rank}\left(A\right)\ge n-\text{rank}\left(B\right)$
3. $\text{rank}\left(A\right)=\text{rank}\left(B\right)$
4. $\text{nullity}\left(A\right)+\text{nullity}\left(B\right)\le n$

73. Which of the following sets of functions are uncountable ?

1. $\{f \mid f:\mathbb{N}\to\mathbb{Q}\}$
2. $\{f \mid f:\{1,2\}\to\mathbb{N}\}$
3. $\{f \mid f:\{1,2\}\to\mathbb{Q}\}$
4. $\{f \mid f:\mathbb{Q}\to\{1,2\}\}$

74. Let $\langle x_n\rangle$ be a sequence of real numbers then which of the following is/are true :

1. If $\langle x_n\rangle$ is unbounded then there exists a subsequence $\langle x_{n_k}\rangle$ such that $\left|x_{n_k}\right|\ge k$ for each $k\in\mathbb{N}$
2. If there exists a subsequence $\langle x_{n_k}\rangle$ such that $\left|x_{n_k}\right|\ge k$ for each $k\in\mathbb{N}$ then $\langle x_n\rangle$ is unbounded.
3. Both are true.
4. Both are false.

75. Which of the following sets in $\mathbb{R}^2$ have positive Lebesgue measure ?

$\left[\begin{array}{l}\text{For two sets } A, B \subseteq \mathbb{R}^2, A+B \\ \quad = \{a+b \mid a \in A, b \in B\}\end{array}\right]$

1. $S = \{(x, y) \mid x^2 + y^2 = 1\}$
2. $S = \{(x, y) \mid x^2 + y^2 < 1\}$
3. $S = \{(x, y) \mid x = y\} + \{(x, y) \mid x = -y\}$
4. $S = \{(x, y) \mid x = y\} + \{(x, y) \mid x = y\}$

76. Let $p_n(x) = x^n$ for $x \in \mathbb{R}$ and let $W = \operatorname{span}\{p_0, p_1, p_2, ...\}$. Then

1. W is the vector space of all real valued continuous functions on $\mathbb{R}$.
2. W is a subspace of all real valued continuous functions on $\mathbb{R}$.
3. $\{p_0, p_1, p_2, ...\}$ is a linearly independent set in the vector space of all continuous functions on $\mathbb{R}$.
4. Trigonometric functions belong to W.

77. Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a continuous function such that $\int_{\mathbb{R}^n} |f(x)| dx < \infty$.

Let A be a real $n \times n$ invertible matrix and for $x, y \in \mathbb{R}^n$, let $\langle x, y \rangle$ denote the standard inner product in $\mathbb{R}^n$. Then

$\int_{\mathbb{R}^n} f(Ax) e^{i\langle y, x\rangle} dx =$

1. $\int_{\mathbb{R}^n} f(x) e^{i\langle (A^{-1})^T y, x\rangle} \frac{dx}{|\det A|}$
2. $\int_{\mathbb{R}^n} f(x) e^{i\langle A^T y, x\rangle} \frac{dx}{|\det A|}$
3. $\int_{\mathbb{R}^n} f(x) e^{i\langle (A^T)^{-1} y, x\rangle} dx$
4. $\int_{\mathbb{R}^n} f(x) e^{i\langle A^{-1} y, x\rangle} \frac{dx}{|\det A|}$

78. Let $\{a_0, a_1, a_2, ...\}$ be a sequence of real numbers. For any $k \geq 1$, let $S_n = \sum_{k=0}^{n} a_{2k}$.

Which of the following statements are correct ?

1. If $\lim_{n\to\infty} S_n$ exists, then $\sum_{m=0}^{\infty} a_m$ exists.
2. If $\lim_{n\to\infty} S_n$ exists, then $\sum_{m=0}^{\infty} a_m$ need not exist.
3. If $\sum_{m=0}^{\infty} a_m$ exists, then $\lim_{n\to\infty} S_n$ exists.
4. If $\sum_{m=0}^{\infty} a_m$ exists, then $\lim_{n\to\infty} S_n$ need not exist.

Unit – II

79. Consider the function

$$f(z) = \frac{\sin(\pi z) e^{\frac{1}{z-4}}}{(z^2 - 1)\cos\left(\pi\left(z + \frac{1}{2}\right)\right)}$$ then which of the following is/are not true :

1. $z = 1$ is a simple pole.
2. $z = -1$ is a removable singularity.
3. $z = 4$ is a non isolated essential singularity.
4. Every prime natural number is a removable singularity.

80. Let $f : \mathbb{C} \to \mathbb{C}$ be an entire function then which of the following is/are not true :

1. $\mathbb{C} - f(\mathbb{C})$ is atmost a singleton set.
2. $\mathbb{C} - f(\mathbb{C})$ is a singleton set.
3. $\mathbb{C} - f(\mathbb{C})$ is empty.
4. $\mathbb{C} - f(\mathbb{C})$ is uncountable.

81. Let $a,b,c \in \mathbb{Q}$ such that

$\dfrac{1+\sqrt[3]{4}}{2-\sqrt[3]{2}} = a + b\sqrt[3]{2} + c\sqrt[3]{4}$ then which of the following is/are not true :

1. $a+b+c = \dfrac{17}{6}$
2. $a+b+c \neq \dfrac{17}{6}$
3. $a \cdot b \cdot c = \dfrac{20}{27}$
4. $a \cdot b \cdot c \neq \dfrac{20}{27}$

82. Let p be a polynomial in 1-complex variable. Suppose all zeros of p are in the upper half plane $H = \{z \in \mathbb{C} | \operatorname{Im}(z) > 0\}$. Then

1. $\operatorname{Im}\dfrac{p'(z)}{p(z)} > 0$ for $z \in \mathbb{R}$.
2. $\operatorname{Re} i\dfrac{p'(z)}{p(z)} < 0$ for $z \in \mathbb{R}$.
3. $\operatorname{Im}\dfrac{p'(z)}{p(z)} > 0$ for $z \in \mathbb{C}$, with $\operatorname{Im} z < 0$.
4. $\operatorname{Im}\dfrac{p'(z)}{p(z)} > 0$ for $z \in \mathbb{C}$, with $\operatorname{Im} z > 0$.

83. Which of the following primes satisfy the congruence $a^{24} \equiv 6a + 2 \bmod 13$?

1. 41
2. 47
3. 67
4. 83

84. In the ring $\mathbb{Z}_5[x]$ which of the following is/are not true :

1. $3x+2$ and $4x+1$ are associate of each other.
2. $x+4$ and $2x+3$ are not associate of each other.
3. $3x^2+4x+3 = (3x+2)(x+4)$

   $= (4x+1)(2x+3)$
4. $\mathbb{Z}_5[x]$ is not a unique factorization domain because $3x^2+4x+3$ can be factorized in two different ways.

85. Consider the set $\mathbb{Z}$ of integers, with the topology τ in which a subset is closed if and only if it is empty, or $\mathbb{Z}$, or finite. Which of the following statements are true ?

1. τ is the subspace topology induced from the usual topology on $\mathbb{R}$.
2. $\mathbb{Z}$ is compact in the topology τ.
3. $\mathbb{Z}$ is Hausdroff in the topology τ.
4. Every infinite subset of $\mathbb{Z}$ is dense in the topology τ.

86. Let G be a group of order 49 then which of the following is/are not true :

1. either G is cyclic or $a^7 = e$ for all $a \in G$
2. G is abelian
3. G is cyclic
4. G has a non trivial proper normal subgroup.

87. Determine which of the following polynomials are irreducible over the indicated rings.

1. $x^5 - 13x^4 + 7x^3 - 6x + 1$ over $\mathbb{R}$
2. $x^3 + 2x^2 + x + 1$ over $\mathbb{Q}$

3. $x^3+15x^2-10x+10$ over $\mathbb{Z}$

4. x^4+7x^2+1 over $\mathbb{Z}$

88. Which of the following groups are non-cyclic :

1. $(\mathbb{Q},+)$ 2. $(\mathbb{Q}^*,\cdot)$

3. $(\mathbb{Q}^+,\cdot)$ 4. $(\mathbb{R}^+,\cdot)$

89. Which of the following is/are not true :

1. In a finite commutative ring every prime ideal is maximal ideal
2. In a commutative ring every prime ideal is maximal ideal
3. In an integral domain every prime ideal is a maximal ideal
4. In a unique factorization domain every prime ideal is maximal ideal

90. Consider the following subsets of the complex plane :

$$\Omega_1=\left\{C\in\mathbb{C}:\begin{bmatrix}1 & C\\ \bar{C} & 1\end{bmatrix}\text{is non}-\text{negative definite } (\text{or equivalently positive semi}-\text{definite})\right\}$$

$$\Omega_2=\left\{C\in\mathbb{C}:\begin{bmatrix}1 & C & C\\ \bar{C} & 1 & C\\ \bar{C} & \bar{C} & 1\end{bmatrix}\text{is non}-\text{negative definite}\begin{pmatrix}\text{or equivalently positive}\\ \text{semi}-\text{definite}\end{pmatrix}\right\}$$

Let $\bar{D}=\{z\in\mathbb{C}\,|\,|z|\le 1\}$. Then

1. $\Omega_1=\bar{D},\ \Omega_2=\bar{D}$
2. $\Omega_1\ne\bar{D},\ \Omega_2=\bar{D}$
3. $\Omega_1=\bar{D},\ \Omega_2\ne\bar{D}$
4. $\Omega_1\ne\bar{D},\ \Omega_2\ne\bar{D}$

Unit - III

91. The initial value problem

$\frac{dy}{dx}=\sqrt{y},\ y>0,\ y(0)=\alpha,\ \alpha\ge 0$ has

1. at least two solutions if $\alpha=0$
2. no solution if $\alpha>0$
3. at least one solution if $\alpha>0$
4. a unique solution if $\alpha=0$

92. Let $f,g:[-1,1]\to\mathbb{R}$,

$f(x)=x^3,\ g(x)=x^2|x|$. Then which is not true :

1. f and g are linearly independent on $[-1,1]$
2. f and g are linearly dependent on $[-1,1]$
3. $f(x)g'(x)-f'(x)g(x)$ is not identically zero on $[-1,1]$
4. $\exists$ a continuous function $p(x)$ and $q(x)$ such that f and g satisfy $y''+py'+qy=0$ on $[-1,1]$

93. Let $y(x)$ be the solution of the differential equation $\frac{d}{dx}\left(x\frac{dy}{dx}\right)=x$,

$y(1)=0,\ \left(\frac{dy}{dx}\right)_{x=1}=0$. Then the value of $y(2)$ is not equal to

1. $\frac{3}{2}+\frac{1}{2}\log 2$ 2. $\frac{3}{2}-\frac{1}{2}\log 2$

3. $\frac{3}{2}+\log 2$ 4. $\frac{3}{2}-\log 2$

94. Let u be a solution of the heat equation

$u_t-u_{xx}=0,\ 0<x<\pi$ and $t>0$

$u(0,t)=u(\pi,t)=0,\ t>0$

$u(x,0)=\sin x(1+2\cos x),\ 0\le x\le\pi$.

Then

1. $u(x,t) \to 0$ as $t \to \infty$ for all $x \in (0,\pi)$
2. $t^2 u(x,t) \to 0$ as $t \to \infty$ for all $x \in (0,\pi)$
3. $e^2 u(x,t)$ is bounded function for $x \in (0,\pi)$, $t > 0$
4. $e^{2t} u(x,t) \to 0$ as $t \to \infty$ for all $x \in (0,\pi)$

95. Pick the region in which the following differential is hyperbolic

$yu_{xx} + 2xyu_{xy} + xu_{yy} = u_x$

1. $xy \neq 1$ 2. $xy \neq 0$
3. $xy > 1$ 4. $xy > 0$

96. Solution of the PDE

$\left(D^2 - D'^2 + D + 3D' - 2\right)z = 0$ is

1. $z = e^{-2x}\phi_1(y+x) + xe^x\phi_2(y+x)$
2. $z = e^{-2x}\phi_1(y+x) + e^x\phi_2(y-x)$
3. $z = e^{-2x}\phi_1(y-x) + xe^x\phi_2(y-x)$
4. Out of all one is correct

97. Given $y'' + xy' + y = 0,\ y(0) = 1,\ y'(0) = 0$ then

1. the equivalent integral equation is a Volterra equation of first kind
2. the equivalent integral equation is Fredholm equation
3. $y(x) = -1 - \int_0^x (2x - \xi) y(\xi) d\xi$ is the equivalent integral equation
4. the equivalent integral equation is a Volterra equation of second kind

98. Given the integral equation

$\int_0^x y(\xi)(x-\xi)^{\frac{-1}{2}} d\xi = 1$ is

1. Volterra integral equation of first kind
2. Volterra integral equation of second kind
3. Fredholm integral equation
4. the solution of the equation is $y(x) = \dfrac{1}{\pi\sqrt{x}}$

99. For a differentiable function $f : \mathbb{R} \to \mathbb{R}$ define the difference quotient

$(D_x f)(h) = \dfrac{f(x+h) - f(x)}{h};\ h > 0.$

Consider numbers of the form $\hat{h} = h(1+\varepsilon)$ for a fixed $\varepsilon > 0$ and let

$e_1(h) = f'(x) - (D_x f)(h),$
$e_2(h) = (D_x f)(h) - (D_x f)(\hat{h}),$
$e(h) = e_1(h) + e_2(h).$

If $f(x + \hat{h}) = f(x+h)$, then

1. $e_1(h) \to 0$ as $h \to 0.$
2. $e_2(h) \to 0$ as $h \to 0.$
3. $e_2(h) \to \varepsilon f'(x)/(1+\varepsilon)$ as $h \to 0.$
4. $e(h) \to 0$ as $h \to 0.$

100. Let y_n satisfy $y_n = y_{n-1} + hy_{n-1}$ with $y_0 = 1 (n = 1, 2, ..., N)$ and for $0 < h < 1$, $Nh = 1$. Then

1. $y_N \to e$ as $N \to \infty$
2. $y_N \to e^h$ as $N \to \infty$
3. $y_n = (1+h)^n$
4. $y_n \geq 1$

101. Let q_α and p_α $(\alpha = 1, 2, ..., n)$ be the generalized coordinates and the generalized momenta, respectively. If H denotes the Hamiltonian and q_a (for some $\alpha = \alpha_0$) is an ignorable coordinate, then which of the following equations are satisfied ?

1. $p_\alpha = -\frac{\partial H}{\partial q_\alpha}, q_\alpha = \frac{\partial H}{\partial p_\alpha}, \forall \alpha$
2. $p_\alpha = \frac{\partial H}{\partial q_\alpha}, q_\alpha = -\frac{\partial H}{\partial p_\alpha}, \forall \alpha$
3. $p_{\alpha_0} = 0, \; q_{\alpha_0} = \frac{\partial H}{\partial p_{\alpha_0}}$
4. $p_{\alpha_0} = \frac{\partial H}{\partial q_{\alpha_0}}, \; q_{\alpha_0} = 0$

102. For a conservative system, the end configurations are fixed and the velocity in the varied motion is such that $T + V = E$. Here T, V and E represent, respectively the kinetic energy, the potential energy and the total energy. If $\delta(A)$ denotes the infinitesimal change in a variable A, and p_α and q_α $(\alpha = 1, 2, ..., n)$ represent the generalized momenta and generalized coordinates, respectively, then

1. $\delta \int T \, dt = 0$
2. $\delta \int \sum_{\alpha=1}^{n} p_\alpha dq_\alpha = 0$
3. $\delta \int \sum_{\alpha=1}^{n} q_\alpha dp_\alpha = 0$
4. $\delta \int \sum_{\alpha=1}^{n} (p_\alpha dq_\alpha + q_\alpha dp_\alpha) = 0$

Unit - IV

103. Let $c \in \mathbb{R}$ be a constant. Let X, Y be random variables with joint probability density function

$$f(x, y) = \begin{cases} cxy, & \text{if } 0 < x < y < 1, \\ 0, & \text{otherwise} \end{cases}$$

Which of the following statements are correct ?

1. $c = \frac{1}{8}$
2. $c = 8$
3. X and Y are independent
4. $P(X = Y) = 0$

104. Let $\{X_n, n \geq 1\}$ be i.i.d. uniform $(-1, 2)$ random variables. Which of the following statements are true ?

1. $\frac{1}{n}\sum_{i=1}^{n} X_i \to 0$ almost surely
2. $\left\{\frac{1}{2n}\sum_{i=1}^{n} X_{2i} - \frac{1}{2n}\sum_{i=1}^{n} X_{2i-1}\right\} \to 0$ almost surely
3. $\sup\{X_1, X_2, ...\} = 2$ almost surely
4. $\inf\{X_1, X_2, ...\} = -1$ almost surely

105. Let $\{X_n\}$ be a Markov chain on $\{0, 1, 2, ...\}$ with

$P_{00} = \frac{2}{3}, \; P_{01} = \frac{1}{3}, \; P_{i,i+1} = \frac{2}{3}, \; P_{i,i-1} = \frac{1}{3},$

$i \geq 1, \; P_{ij} = 0$ otherwise.

Which of the following statements are correct ?

1. $\{X_n\}$ is recurrent
2. $\{X_n\}$ is transient

3. $P\left(\lim_{n\to\infty} X_n = 0\right) > 0$

4. $P\left(\lim_{n\to\infty} X_n = +\infty\right) > 0$

106. Which of the following statements are correct ?

1. For a finite state Markov chain there is at least one transient state.
2. For a finite state Markov chain there is at least one stationary distribution.
3. For a countable state Markov chain, every state can be transient.
4. For an aperiodic countable state Markov chain there is at least one stationary distribution.

107. Suppose X follows an exponential distribution with parameter $\lambda > 0$. Fix $a > 0$. Define the random variable Y by $Y = k$, if $ka \le X < (k+1)a$, $k = 0,1,2,....$

Which of the following statements are correct ?

1. $P(4 < Y < 5) = 0$
2. Y follows an exponential distribution
3. Y follows a geometric distribution
4. Y follows a Poisson distribution

108. Let $\{X_1,....,X_n\}$ be a random sample from the probability density function $f(x;\theta) = \frac{1}{2}e^{-|x-\theta|}; -\infty < x < \infty$ where $\theta \in \mathbb{R}$.

Which of the following statements are correct ?

1. The maximum likelihood estimator of θ is $\frac{1}{n}\sum_{i=1}^{n} X_i$.
2. $\sum_{i=1}^{n} X_i$ is a sufficient statistic for θ
3. The maximum likelihood estimator of θ is a function of a sufficient statistic.
4. There does not exist a *uniformly most powerful* test for the following testing problem : $H_0 : \theta = 0$ vs $H_1 : \theta \ne 0$

109. Consider the problem of testing $H_0 : \theta = 1$ vs $H_1 : \theta = \frac{1}{2}$ where θ is the mean of a Poisson random variable. Let X and Y be a random sample from Poisson (θ) distribution. Consider the following test procedure :

Reject H_0 if either $X = 0$ or ($X = 1$ and $X + Y \le 2$); otherwise accept H_0.

Which of the following are true ?

1. P [type I error] $= e^{-1} + 2e^{-2}$
2. P [type II error] $= 1 - \frac{1}{2}e^{-1} - e^{-\frac{1}{2}}$
3. Size of the test $e^{-1} + e^{-2}$
4. Power of the test is $\frac{3}{4}e^{-1} + e^{-\frac{1}{2}}$

110. Suppose $\{X_1,....,X_n\}$ is a random sample from the distribution with probability density testing problem using likelihood ratio test (LRT).

$$H_0: f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \text{ vs}$$

$$H_1: f(x) = \frac{1}{2} e^{-|x|}$$

Which of the following statements are correct ?

1. There does not exist any LRT.
2. The rejection region is a function of $|X_1|,....,|X_n|$
3. The rejection region is a function of $X_1^2,....,X_n^2$.
4. The rejection region is of the form $\left\{\sum_{i=1}^{n}(|X_i|-1)^2 \geq c\right\}$

111. Let $X_1, X_2,...., X_7$ be i.i.d. random variables with common continuous distribution function $F(x-\theta_1)$ and let $Y_1, Y_2,..., Y_7$ be i.i.d. random variables with common continuous distribution function $F(y-\theta_2)$. Consider the problem of testing $H_0: \theta_1 = \theta_2$ vs $H_1: \theta_1 > \theta_2$.

Let $R_1, R_2,...., R_7, R_8, R_9,..., R_{14}$ be the ranks of $X_1, X_2,..., X_7, Y_1, Y_2,..., Y_7$, respectively in the combined sample. Define

$$T_1 = \sum_{i=1}^{7} R_i \quad \text{and} \quad T_2 = \sum_{j=8}^{14} R_j .$$

Which of the following statements are true?

1. $E(T_1) = E(T_2)$ under H_0
2. $E(T_1) = 52.5$ under H_0
3. T_2 cannot be 27
4. If we use right-tailed test based on T_1, then the observed value $T_1 = 77$ is significant at 5% level of significance.

112. In a football league, the goals scored by home teams over 380 matches have the following frequency distribution.

Number of goals	0	1	2	3	4	5
Frequency	92	121	91	50	19	7

The average goals scored by home teams is 1.49. We want to test

H_0: Goal distribution is Poisson.

Based on observations the value of the χ^2 – statistic for goodness of fit is 1.27.

Given $\chi^2_{0.05,6} = 1.64, \chi^2_{0.05,5} = 1.15,$ $\chi^2_{0.95,6} = 12.59$ and $\chi^2_{0.95,5} = 11.07,$ which of the following are true ?

1. H_0 is not rejected at 5% level of significance.
2. χ^2 – statistic has 5 degrees of freedom
3. Under H_0, the maximum likelihood estimate (MLE) of the rate parameter of Poisson is 1.49
4. Under H_0, in a game, the MLE of the probability that home team will score at most one goal is $2.49e^{-1.49}$

113. Consider the model $Y = X\beta + \varepsilon,$ where

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix},\ X = \left(\left(x_{ij}\right)\right)_{n\times p} \text{ and } \beta = \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_p \end{bmatrix},$$

$$\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}.$$

$E(\varepsilon) = 0$ and $D(\varepsilon) = \sigma^2 I_n, p < n.$ Let $\hat{\beta}$ be the solution of $X^T X\beta = X^T Y$. Which of the following are true ?

1. If $C^T\beta$ is estimable then $C^T\hat{\beta}$ is the best linear unbiased estimator (BLUE) of $C^T\beta$.
2. All linear parametric functions are estimable if and only if $\text{Rank}(X) > p$.
3. If $\text{Rank}(X) < p$ then some linear parametric functions are not estimable.
4. $(Y - X\hat{\beta})^T (Y - X\hat{\beta})/(n-p)$ is an unbiased estimator of σ^2.

114. Let X_1 and X_2 be independent random variables each having $N(\mu,\sigma^2)$ distribution, where $\mu \in \mathbb{R}, \sigma^2 > 0$. Let $0 \le \theta < 2\pi$ and $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$. Put $\underline{Y} = (Y_1, Y_2)^t = A\underline{X}$ with $\underline{X} = (X_1, X_2)^t$.

Which of the following statements are correct ?

1. $\underline{Y} = \underline{X}$ in distribution if and only if $\mu = 0$.
2. $\underline{Y} = \underline{X}$ in distribution if and only if $\mu = 0, \theta = 0$
3. Y_1 and Y_2 are Gaussian.
4. Y_1 and Y_2 may be correlated.

115. Let X_1 and X_2 be two i.i.d. $N_p(0,\sum)$ random variables with $\text{rank}(\sum) = p$. Suppose A is a $p\times p$ symmetric matrix of rank r, and $A^2 = A$. Which of the following statements are correct ?

1. $X_1^T A X_1 \sim X_r^2$
2. $X_1^T A X_1 + X_2^T A X_2 \sim 2X_r^2$
3. $X_1^T A X_2 + X_2^T A X_1 \sim 2X_r^2$
4. $X_1^T A X_1 + X_2^T A X_2 \sim X_{2r}^2$

116. Consider a finite population of size N. Let T_1 be the sample mean based on a sample of size n under simple random sampling with replacement (SRSWR) scheme. Let T_2 be the sample mean based on a stratified random sample of size n where the samples are drawn from each of 4 strata using SRSWR scheme under proportional allocation. Then which of the following are sufficient conditions for $\text{Var}(T_1) = \text{Var}(T_2)$ to hold ?

1. Strata sizes are same
2. Strata totals are same
3. Strata means are same
4. Strata variances are same

117. Consider a balanced incomplete block design (BIBD) with parameters b, v, r, k, λ where each of the *b* blocks contains *k* treatments out of a set of *v* treatments, each treatment occurs *r* times in the design and each pair of treatments occurs λ times. A new design is formed by replacing all the treatments in each block by its complementary set. Then which of the following are true for the new design ?
1. It is a BIDB
2. Each treatment occurs $(b-r)$ times
3. Each pair of treatments appears in the same block $(b-r+\lambda)$ number of times
4. $bk = vr$

118. Suppose the random variable *X* has the following probability density function

$$f(x) = \begin{cases} \alpha(x-\mu)^{\alpha-1} e^{-(x-\mu)^{\alpha}}; & x > \mu \\ 0 & x \le \mu, \end{cases}$$

where $\alpha > 0, -\infty < \mu < \infty$. Which of the following statements are correct ? the hazard function of *X* is
1. an increasing function for all $\alpha > 0$
2. a decreasing function for all $\alpha > 0$
3. an increasing function for some $\alpha > 0$
4. a decreasing function for some $\alpha > 0$

119. Consider the problem :

Maximize $2y_1 + 3y_2 + 5y_3 + 4y_4$

subject to

$y_1 + y_2 \le 1,\ y_2 + y_3 \le 1,\ y_3 + y_4 \le 1,$

$y_4 + y_1 \le 1$ and $y_i \ge 0$ for $i = 1, 2, 3, 4.$

Then the optimum value is
1. equal to 8
2. between 8 and 9
3. greater than or equal to 7
4. less than or equal to 7

120. Let (x_1, x_2, x_3, x_4) be an optimal solution to the problem of minimizing

$x_1 + x_2 + x_3 + x_4$

Subject to the constraints

$x_1 + x_2 \ge 300$
$x_2 + x_3 \ge 500$
$x_3 + x_4 \ge 400$
$x_4 + x_1 \ge 200$
$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0,\ x_4 \ge 0.$

Which of the following are not possible values for any x_i ?

1. 300 2. 400
3. 500 4. 600

Answer Key

Part - A		**Part - B**				**Part - C**			
1.	3	21.	1	51.	1	61.	2,4	91.	1,3
2.	3	22.	4	52.	1	62.	1,2	92.	1,3,4
3.	1	23.	2	53.	3	63.	1,2,3,4	93.	1,2,3,4
4.	2	24.	3	54.	2	64.	2	94.	1,2,3
5.	1	25.	4	55.	4	65.	3,4	95.	3
6.	4	26.	3	56.	2/3/4	66.	1,2,3,4	96.	2,4
7.	3	27.	3	57.	4	67.	1,2	97.	3,4
8.	4	28.	1	58.	4	68.	1,2,3	98.	1,4
9.	2	29.	2	59.	1	69.	1,2,3,4	99.	1,3
10.	3	30.	2	60.	3	70.	3,4	100.	1,3,4
11.	1	31.	4			71.	2,4	101.	1,3
12.	2	32.	4			72.	1,2,4	102.	1,2
13.	1	33.	2			73.	1,4	103.	2,4
14.	1	34.	1			74.	1,2,3	104.	2,3,4
15.	2	35.	3			75.	2,3	105.	2,4
16.	3	36.	4			76.	2,3	106.	2,3
17.	1	37.	2			77.	1	107.	1,3
18.	4	38.	4			78.	2,4	108.	3,4
19.	3	39.	3			79.	2,3	109.	1,4
20.	4	40.	4			80.	1,2,3,4	110.	2,3,4
		41.	1			81.	2,4	111.	1,2,3,4
		42.	2			82.	1,2,3	112.	1,2,3,4
		43.	3			83.	1,3	113.	1,3
		44.	1			84.	2,4	114.	3
		45.	1			85.	2,4	115.	1,4
		46.	4			86.	3	116.	3
		47.	1			87.	2,3,4	117.	1,2,4
		48.	*			88.	1,2,3,4	118.	3,4
		49.	3			89.	1,2,3,4	119.	3,4
		50.	4			90.	3	120.	2,3,4