# RISING ★ STAR ACADEMY

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## INSTRUCTIONS

1. You have opted for English as medium of question paper. This test Booklet contains one hundred and twenty (20 Part' A' +40 Part 'B' +60 Part 'C') Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 25 and 20 questions from part 'A' 'B' and 'C' respectively. If more than required number of questions are answered, only first 15, 25 and 20 questions in Parts 'A' 'B' and 'C' respectively, will be taken up for evaluation.
2. Each question in Part 'A' carries 2 marks, Part 'B' 3 marks and Part 'C' 4.75 marks respectively. There will be negative marking @0.5 marks in Part 'A' and @ 0.75 marks in Part 'B' for each wrong answer and no negative marking for Part 'C'.
3. Below each question in Part 'A' and 'B', four alternatives or responses are given. Only one of these alternatives is the "correct" option to the question. You have to find, for each question, the correct or the best answer. In Part 'C' each question may have "**ONE**' or '**MORE**' correct options in Part 'C'. Credit in a question shall be given only on idenfication of 'ALL' the correct options in Part 'C'. No credit shall be allowed in a question if any incorrect option is marked as correct answer.

## Part – A

1. If the fraction $\frac{m}{n}$ is negative, which of the following cannot be true ?

   1. $\frac{n}{m} > \frac{m}{n}$  2. $mn < 0$
   3. $(n-m) < 0$  4. $mn^3 > 0$

2. Out of 9 persons, 8 persons spent Rs. 30 each for their meals. The ninth one spent Rs. 20 more than the average expenditure of all the nine. The total money spent by all of them was :

   1. Rs. 260  2. Rs. 290
   3. Rs. 292.50  4. Rs. 400.50

3. Two pipes A and B can fill a tank in 6 hours respectively. If they are opened on alternate hours and if pipe A is opened first, in how many hours, the tank shall be full ?

   1. 4  2. $4\frac{1}{2}$
   3. 5  4. $5\frac{1}{2}$

4. The cost price of 20 articles is the same as the selling price of *x* articles : If the profit is 25%, then the value of *x* is :

   1. 15  2. 16
   3. 18  4. 25

5. A train running at 54 kmph takes 20 seconds to pass a platform. Next it takes 12 seconds to pass a man walking at 6 kmph in the same direction in which the train is going. Find the length of the train and the length of the platform.
   1. 140m　　2. 160m
   3. 150m　　4. 170m
6. The volume and whole surface area of a cylindrical solid of radius '$r$' units are V and S respectively. If the height of the cylinder is 1 unit, then $\frac{V}{S}$ is equal to :
   1. $\frac{1}{2}\left(1-\frac{1}{r+1}\right)$　　2. $\frac{1}{2}\left(1+\frac{1}{r+1}\right)$
   3. $\frac{1}{2}\left(1-\frac{1}{r}\right)$　　4. $\frac{1}{2}\left(1+\frac{1}{r}\right)$
7. Two circles of radius 4cm and 6cm touch other internally. Find the longest chord of bigger circle which is outside of the smaller circle ?
   1. $8\sqrt{2}$ cm　　2. $4\sqrt{2}$ cm
   3. $6\sqrt{2}$ cm　　4. $3\sqrt{2}$ cm
8. The tangent drawn at P and Q on the circumference of a circle intersect at A. If $\angle PAQ = 68^\circ$, the measure of the $\angle APQ$ is
   1. $56^\circ$　　2. $68^\circ$
   3. $28^\circ$　　4. $34^\circ$
9. Ram starts walking straight towards east. After walking 75m he turns to the left and walks 25m straight. Again he twins to the left and walks a distance of 40m straight, again he turns to the left and walks a distance of 25m. How far is he from the starting point ?
   1. 140m　　2. 25m
   3. 35m　　4. 115m
10. Five persons were playing card game sitting in a circle all facing the centre. Ashish was to the left of Milan, Nitin was to the right of Anupam and between Anupam and Mukesh. Who was to the right of Mukesh.
    1. Milan　　2. Nitin
    3. Mukesh　　4. Ashish
11. If it was Monday on January 1, 2006. What was the day of the week on January, 2008.
    1. Thursday　　2. Wednesday
    3. Monday　　4. Tuesday
12. Two clocks are set at 1 pm. Fast clock gains 1 min for every hour. Find the time when the fast clock shows 6 pm.
    1. 5 : 00 hrs.　　2. $5:55\frac{5}{61}$ hrs.
    3. 4 : 56 hrs.　　4. $4:55\frac{4}{11}$ hrs.
13. Two players play the following game : A writes 3, 5, 6 on three different cards ; B writes 8, 9, 10 on three different cards. Both draw randomly two cards from their collections. Then A computes the product of two numbers he/she has drawn, and B computes the sum of two numbers he/she has drawn. The player getting the larger number wins. What is the probability that A wins ?
    1. $\frac{1}{3}$　　2. $\frac{5}{9}$

3. $\frac{4}{9}$ 4. $\frac{1}{9}$

14. The average of six numbers is $x$ and the average of three of these is $y$. If the average of the remaining three is $z$, than :
   1. $x = y + z$ 2. $2x = y + z$
   3. $x = 2y + 2z$ 4. None of these

15. OPI, ?, ODP, MBQ, IAW
   1. RHJ 2. SHJ
   3. SIJ 4. TIJ

16. A towel, when bleached, was found to have last 20% of its length and 10% of its breadth. The percentage of decrease in area is :
   1. 10% 2. 10.08%
   3. 20% 4. 28%

17. Walking $\frac{6}{7}$th of his usual speed, a man is 12 minutes too late. The usual time taken by him to cover that distance is :
   1. 1 hour 2. 1 hr 12 min
   3. 1 hr 15 min 4. 1 hr 20 min

18. The present worth of Rs. 169 due to 2 years at 4% per annum compound interest annually, is :
   1. Rs. 150.50 2. Rs. 154.75
   3. Rs. 156.25 4. Rs. 158

19. A sum of Rs. 53 is divided among A, B, C in such a way that A gets Rs. 7 more than what B gets and B gets Rs. 8 more than what C gets. The ratio of their shares is :
   1. $16:9:18$ 2. $25:18:10$
   3. $18:25:10$ 4. $15:8:30$

20. The average annual income (in Rs.) of certain agricultural workers is S and that of other workers is T. The number of agricultural workers is 11 times that of other workers. Then the average monthly income (in Rs.) of all the workers is :
   1. $\frac{S+T}{2}$ 2. $\frac{S+11T}{2}$
   3. $\frac{1}{11S}+T$ 4. $\frac{11S+T}{12}$

**Part - B**

**Unit – I**

21. The determinant $\begin{vmatrix} 1 & 1+x & 1+x+x^2 \\ 1 & 1+y & 1+y+y^2 \\ 1 & 1+z & 1+z+z^2 \end{vmatrix}$ is equal to
   1. $(z-y)(z-x)(y-x)$
   2. $(x-y)(x-z)(y-z)$
   3. $(x-y)^2(y-z)^2(z-x)^2$
   4. $(x^2-y^2)(y^2-z^2)(z^2-x^2)$

22. Let $A$ and $B$ are $n \times n$ real matrices such that $AB = BA = 0$ and $A+B$ is invertible then which of the following is not true :
   1. $\text{Rank}(A) = \text{Rank}(B)$
   2. $\text{Rank}(A) = \text{Rank}(B) = n$
   3. $\text{Nullity}(A) = \text{Nullity}(B) = n$
   4. $A-B$ is invertible

23. Let $A$ is a $m \times n$ matrix. Let $\exists\, b \in \mathbb{R}^m$ such that the system $Ax = b$ has unique solution then
    1. Rows of $A$ are linearly independent
    2. Rows of $A$ are linearly dependent
    3. Columns of $A$ are linearly independent
    4. Columns of $A$ are linearly dependent

24. Let $T : P_3(\mathbb{R}) \to P_3(\mathbb{R})$ be a linear transformation given by $T(p(x)) = p(x) + p'(x) + p''(x)$ then
    1. $T$ is nilpotent
    2. $|T| = 0$
    3. $|T| = 1$
    4. $T$ is idempotent

25. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$; suppose $T$ is a transformation on $M_2(\mathbb{R})$ given by $T(X) = AX$ then which of the following is not true :
    1. $|T| = 36$
    2. $\operatorname{tr}(A) = 10$
    3. $|A| = 6$
    4. None of the above

26. Let $V = \{(x_1, x_2, ..., x_n) : x_1 = x_4 = x_9 = ... = 0\} \subseteq \mathbb{R}^n$ be a subspace of $\mathbb{R}^n$ then $\dim(V)$ is
    1. $n - \left[\frac{n}{2}\right]$
    2. $n - \left[\sqrt{n}\right]$
    3. $\frac{n}{2}$
    4. $\sqrt{n}$

27. Let $(a_n) = \frac{(n!)^{\frac{1}{n}}}{n}$ then $\lim_{n \to \infty} a_n$ is
    1. $e$
    2. $\frac{1}{e}$
    3. 1
    4. 0

28. The series $\sum_{n=1}^{\infty} \frac{n}{4^n}$
    1. does not converges
    2. converges to $\frac{4}{3}$
    3. converges to 1
    4. converges to $\frac{4}{9}$

29. Let $E$ be a subset of $\mathbb{R}$ then the characteristic function $\chi_E : \mathbb{R} \to \mathbb{R}$ is continuous if $f$
    1. $E$ is closed
    2. $E$ is open
    3. $E = \mathbb{R}$ or $E = \phi$
    4. None of the above

30. Which of the following functions is not uniformly continuous on (0, 1)
    1. $f(x) = e^x$
    2. $f(x) = x$
    3. $f(x) = \tan \frac{\pi x}{2}$
    4. $f(x) = \sin x$

31. The value of $\int_0^1 \left( \sum_{n=1}^{\infty} \frac{x^n}{n} \right) dx$ is
    1. $\sum_{n=1}^{\infty} \frac{1}{n^2(n+1)}$

2. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$

3. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)^2}$

4. $\sum_{n=1}^{\infty}\frac{1}{(n(n+1))^2}$

32. Let $f:\mathbb{R}^2\to\mathbb{R}^2$ be given by $f(x,y)=(x^2,y^2+\sin x)$ then the derivative of $f$ at $(x,y)$ is the linear transformation given by

1. $\begin{pmatrix}2x & 0\\ \cos x & 2y\end{pmatrix}$ 2. $\begin{pmatrix}2x & 0\\ 2y & \cos x\end{pmatrix}$

3. $\begin{pmatrix}2y & \cos x\\ 2x & 0\end{pmatrix}$ 4. $\begin{pmatrix}2x & 2y\\ 0 & \cos x\end{pmatrix}$

**Unit - II**

33. Which of the following rings is not a principal ideal domain
1. $\mathbb{Z}_7[x]$ 2. $\mathbb{Z}[x]$
3. $\mathbb{Q}[x]$ 4. $\mathbb{R}[x]$

34. The total number of non isomorphic abelian groups of order 202 is
1. 2 2. 1
3. 101 4. 4

35. The number of words that can be formed by permuting the letters of "CINCINATI"
1. 9! 2. 6!
3. 7! 4. None

36. Let $A\subseteq\mathbb{R}^2$ and $X=\mathbb{R}^2\setminus A$ be subsets with subspace topology inherited from the usual topology on $\mathbb{R}^2$. Then
1. A is countable dense implies that X is totally disconnected.
2. A is unbounded implies that X is compact.
3. A is open implies that X is compact.
4. A is countable implies that X is path connected.

37. Let *f*, *g* be meromorphic functions on $\mathbb{C}$. If $f$ has a pole of order $k$ at $z=a$ and $g$ has a pole of order $m$ at $z=a$, where $k<m$ then $f+g$ has
1. a pole of order $k$ at $z=a$
2. a pole of order $m$ at $z=a$
3. a pole of order $k+m$ at $z=a$
4. a pole of order $km$ at $z=a$

38. Let $p(x)$ be a polynomial of the real variable $x$ of degree $k\geq1$. Consider the power series $f(z)=\sum_{n=0}^{\infty}p(n)(2n^2+2n+1)z^n$ where $z$ is a complex variable. Then the radius of convergence of $f(z)$ is
1. 0 2. 1 3. $k$ 4. $\infty$

39. Which of the following is true :
1. Every finite commutative ring with unity is an integral domain.
2. Every commutative ring with unity is an unique factorization domain.
3. Every integral domain is an unique factorization domain.
4. Every finite integral domain is an unique factorization domain.

40. Which of the following is true :
1. $\mathbb{Z}/175\mathbb{Z}$ has exactly 3 distinct ideals.
2. $\mathbb{Z}/175\mathbb{Z}$ has exactly 3 distinct prime ideals.
3. $\mathbb{Z}/175\mathbb{Z}$ has exactly 2 distinct prime ideals.
4. $\mathbb{Z}/175\mathbb{Z}$ has a unique maximal ideal.

**Unit - III**

41. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solutions of $x^2y''-2xy'-4y=0$ for all $x\in[1,10]$. If $W(1)=1$ then $W(3)-W(2)$ is

1. 1 2. 2
3. 3 4. 5

42. The solution of the differential equation $\frac{d^2y}{dx^2} - y = e^x$ satisfying the boundary conditions $y(0) = 0$ and $y'(0) = \frac{3}{2}$ is

1. $y(x) = \sinh x + \frac{x}{2}e^x$
2. $y(x) = \sinh x - \frac{x}{2}e^x$
3. $y(x) = \cosh x + \frac{x}{2}e^x$
4. $y(x) = x\cosh x + \frac{x}{2}e^x$

43. Let $P(x, y)$ be a particular integral of the PDE $z_{xx} - z_y = 2y - x^2$. Then $P(2,3)$ equals

1. 6 2. 12
3. 5 4. 10

44. Complete integral of the PDE

$p^2(xq + p^2) = pqz - q^2(yp + q^2)$

1. $z = ax^2 + by^2 + \frac{p}{q}$
2. $z = ax + by + \frac{a^2 + b^2}{ab}$
3. $z = ax + by + \frac{a^4 + b^4}{ab}$
4. None

45. For the integral equation

$\phi(x) = x + 2\int_0^x \cos(x - \xi)\phi(\xi)d\xi$ value of $\phi(2)$ is

1. $2e^2 + 4$ 2. $2e^2 - 4$
3. $2e^2 + 2$ 4. None

46. For the integral equation

$y(x) = 2x^2 + \int_0^x 4x\,y(\xi)d\xi$ resolvent kernel is

1. $4xe^{\frac{1}{2}(x^2-\xi^2)}$ 2. $4xe^{2(x^2-\xi^2)}$
3. $2xe^{(x^2-\xi^2)}$ 4. None

47. Consider two waves of same angular frequency $\omega$, same angular wave number $k$, same amplitude $a$ traveling in the positive direction of $x-$ axis with the same speed, and with phase difference $\phi$. Then the superposition principle yields a resultant wave with

1. Amplitude $2a$ and phase $\phi$.
2. Amplitude $2a$ and phase $(\phi/2)$.
3. Amplitude $2a\cos(\phi/2)$ and phase $(\phi/2)$.
4. Amplitude $2a\cos(\phi/2)$ and phase $\phi$.

48. Let $f(x) = ax + b$ for $a, b \in \mathbb{R}$. Then the iteration $x_{n+1} = f(x_n)$ starting from any given $x_0$ for $n \geq 0$ converges

1. for all $a \in \mathbb{R}$
2. for no $a \in \mathbb{R}$
3. for $a \in [0,1)$
4. only for $a = 0$

## Unit - IV

49. Let $Y_1, Y_2, Y_3$ be uncorrelated random variables with common unknown variance $\sigma^2$ and expectations given by

$E(Y_1) = \beta_0 + \beta_1$,

$E(Y_2) = \beta_0 + \beta_2$,

$E(Y_3) = \beta_0 + \beta_3$,

where $\beta_0, \beta_1, \beta_2, \beta_3$ are unknown parameters. Which of the following statements is true ?

1. The degrees of freedom associated with the error sum of squares is 1.
2. An unbiased estimator of $\sigma^2$ is

$\frac{1}{6}\left[\left(Y_1-Y_2\right)^2+\left(Y_1-Y_3\right)^2+\left(Y_2-Y_3\right)^2\right]$.

3. $\beta_0, \beta_1, \beta_2,$ and $\beta_3$ are each individually estimable.
4. $\beta_1-2\beta_2+\beta_3$ is estimable.

50. Let X be a $p\times 1$ random vector such that $X \sim N_p\left(0, \sum\right)$ where rank$\left(\sum\right)=p$. Which of the following is true ?
1. $E\left(X'\sum^{-1}X\right)=2p, V\left(X'\sum^{-1}X\right)=2p$
2. $E\left(X'\sum^{-1}X\right)=2p, V\left(X'\sum^{-1}X\right)=p$
3. $E\left(X'\sum^{-1}X\right)=p, V\left(X'\sum^{-1}X\right)=p$
4. $E\left(X'\sum^{-1}X\right)=p, V\left(X'\sum^{-1}X\right)=2p$

51. A finite population has 8 units, labeled $u_1, u_2, ...., u_8$ and the value of a study variable for the unit $u_i$ is $Y_i\left(i=1,2,....,8\right)$. Let $\bar{Y}=\left(1/8\right)\sum_{i=1}^{8}Y_i$. A sample of size 4 units is drawn from this population in the following manner : a simple random sample (SRS) of size 2 is drawn from the units $u_2, u_3, ...., u_7$ and the sample so selected is augmented by the units $u_1$ and $u_8$ to get a sample of size 4. Let $\bar{y}$ be the sample mean based on the SRS of size two and let $T=\left(Y_1+6\bar{y}+Y_8\right)/8$. Which of the following statements is true ?
1. T is a biased estimator of $\bar{Y}$.
2. T is unbiased for $\frac{1}{6}\sum_{i=2}^{7}Y_i$
3. T is unbiased for $\bar{Y}$ and $V\left(T\right)=3V\left(\bar{y}\right)/4$.
4. T is unbiased for $\bar{Y}$ and $V\left(T\right)=9V\left(\bar{y}\right)/16$

52. At a doctor's clinic patients arrive at an average rate of 10 per hour. The consultancy time per patient is exponentially distributed with an average of 6 minutes per patient. The doctor does not admit any patient if at any time 10 patients are waiting. Then at the steady state of this M/M/1/R queue the expected number of patients waiting is
1. 0
2. 5
3. 9
4. 10

53. Let T be a statistic whose distribution under the null hypothesis $H_0$ is uniform (0, 1). Let the distribution of T under an alternative hypothesis $H_1$ be triangular distribution with density
$$g(x)=\begin{cases} x, & : \quad 0\le x\le 1 \\ 2-x, & : \quad 0\le x\le 2. \end{cases}$$
Then the power $\beta$ of the most powerful test for testing $H_0$ against the alternative $H_1$ based on the statistic T with size 0.1 satisfies
1. $0<\beta\le 0.5$
2. $0.5<\beta\le 0.55$
3. $0.55<\beta\le 0.7$
4. $0.7<\beta\le 1$

54. Let X be a random variable following a Poisson distribution with parameter $\lambda>0$. To estimate $\lambda^5$, consider an estimator $T=X\left(X-1\right)\left(X-2\right)\left(X-3\right)\left(X-4\right)$. Which of the following statements is true ?
1. T is not unbiased.
2. T is unbiased but not UMVUE.
3. T is UMVUE.
4. UMVUE for $\lambda^5$ does not exist.

55. Let $\left(X_n\right)_{n\ge 0}$ be a Markov chain on the state space $S=\{0,1\}$. Then
1. The chain has a unique stationary distribution.
2. $\mathbb{P}\left(X_n=0 | X_0=0\right)$ converges as $n\to\infty$.
3. The chain may have one recurrent and one transient state.
4. The chain is always irreducible.

56. Suppose $X, Y$ and $Z$ are three independent random variables each with finite variance.

Let $U = X + Z$ and $V = Y + Z$. Suppose U and V have the same distribution. Then

1. X and Y have the same distribution.
2. It is possible to have $Corr(U,V) < 0$.
3. $U + V$ and $U - V$ are always independent.
4. We must have $Corr(U,V) < 1$.

57. Suppose you have a coin with probability $\frac{3}{4}$ of getting a Head. You toss the coin twice independently. Let $\Omega = \{(H,H),(H,T),(T,H),(T,T)\}$ be the sample space.Then it is possible to have an event $E \subseteq \Omega$ such that

1. $\mathbb{P}(E) = \frac{1}{3}$
2. $\mathbb{P}(E) = \frac{1}{9}$
3. $\mathbb{P}(E) = \frac{1}{4}$
4. $\mathbb{P}(E) = \frac{7}{8}$

58. Suppose $X_1, X_2, ..., X_n$ are independent random variables each having a $Bin\left(8, \frac{1}{2}\right)$ distribution. Then $\frac{1}{\sqrt{n}} \sum_{k=1}^{n} (-1)^k X_k$ converges in distribution to

1. $N(0,1)$
2. $N(0,2)$
3. $N(4,2)$
4. $N(4,1)$

59. Let $X_1, X_2, ...., X_n$ be iid with common density $f_\theta(x) = \begin{cases} \theta e^{-\theta}, & x > 0 \\ 0, & x \le 0, \end{cases}$ where $\theta > 0$. For testing $H_0 : \theta = 1$ versus $H_1 : \theta = 2$, let $r_n$ be the power of the most powerful test of size $\alpha = 0.05$ with sample size *n*. Then

1. $r_n$ increases to $1 - \alpha$.
2. $r_n$ may not converge.
3. $r_n$ increases to 1.
4. $r_n$ may not be an increasing sequence.

60. Consider the following three sets of sample observations.

Sample 1: $x_1, x_2, ...., x_n$.

Sample 2: $y_1, y_2, ...., y_m$.

Sample 3: $x_1, x_2, ...., x_n, y_1, y_2, ...., y_m$.

Let $\bar{m}_i, \tilde{m}_i, \hat{m}_i$ and $\sigma_i^2$ denote mean, median, mode and variance respectively of the $i^{th}$ sample for $i = 1, 2, 3$. Assume $\bar{m}_1 = \bar{m}_2$. Which of the following is NOT always true ?

1. $\bar{m}_3 = \bar{m}_1$
2. $\min(\tilde{m}_1, \tilde{m}_2) \le \tilde{m}_3 \le \max(\tilde{m}_1, \tilde{m}_2)$.
3. $\min(\hat{m}_1, \hat{m}_2 \le \hat{m}_3 \le \max \hat{m}_1, \hat{m}_2)$.
4. $\min(\sigma_1^2, \sigma_2^2) \le \sigma_3^2 \le \max(\sigma_1^2, \sigma_2^2)$.

## Part - C

### Unit - I

61. Which of the following sets is/are dense in $\mathbb{R}^2$

1. $\mathbb{Q} \times \mathbb{Q}^c$
2. $\mathbb{Q} \times \mathbb{Q}$
3. $\mathbb{Q} \times \mathbb{R}$
4. $\mathbb{Q}^c \times \mathbb{Q}^c$

62. Which of the following sets has cardinality same as $\mathbb{R}$

1. $P(\mathbb{N})$
2. $P(\mathbb{Q})$
3. $P(\mathbb{Q}^c)$
4. $P(\mathbb{R})$

63. Let $A = \{x \in \mathbb{R} : (x-2)(x-3)(x-4) < 0\}$ then

1. $\text{lub}(A) = 4$
2. $\text{glb}(A) = -4$
3. $\text{lub}(A) = \infty$
4. $\text{glb}(A) = -\infty$

64. If $\{x_n\}$ and $\{y_n\}$ are sequences of real numbers which of the following is/are trues :

1. $\limsup_n (x_n + y_n) \le \limsup_n (x_n) + \limsup_n (y_n)$
2. $\limsup_n (x_n + y_n) \ge \limsup_n (x_n) + \limsup_n (y_n)$
3. $\liminf_n (x_n + y_n) \le \liminf_n (x_n) + \liminf_n (y_n)$
4. $\liminf_n (x_n + y_n) \ge \liminf_n (x_n) + \liminf_n (y_n)$

65. Let $P_n(x) = a_n x^2 + b_n x + c_n$ be a sequence of quadratic polynomial. Let

$\lim_{n\to\infty} P_n(0)$; $\lim_{n\to\infty} P_n(1)$ and $\lim_{n\to\infty} P_n(2)$ exist then

1. $\lim_{n\to\infty} P(5)$ exist
2. $\lim_{n\to\infty} P(7)$ exist
3. $\lim_{n\to\infty} P(-1)$ doesn't exist
4. All of above

66. Which of the following is/are true :

1. $\log\frac{1}{4}+\sum_{n=1}^{\infty}\log\frac{(n+1)(3n+1)}{n(3n+4)}=\log\frac{1}{3}$
2. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)(n+3)}=\frac{1}{18}$
3. $\sum_{n=1}^{\infty}\frac{1}{n(n+3)}=\frac{1}{18}$
4. All of the above

67. Let $f:\mathbb{R}\to\mathbb{R}$ be a monotonic function then

1. $f$ may have removable discontinuity
2. $f$ may have jump discontinuity
3. $f$ has atmost countable discontinuities
4. All of above

68. Which of the following sets is dense in $\mathbb{R}$ :

1. $\{\sin n : n\in\mathbb{Z}\}$
2. $\mathbb{Z}+\sqrt{3}\mathbb{Z}$
3. $\mathbb{Z}+\pi\mathbb{Z}$
4. $\left\{\frac{m}{2^n} : m\in\mathbb{Z},\ n\in\mathbb{N}\right\}$

69. Which of the following is/are convergent

1. $\int_{-\infty}^{0}\frac{dx}{p^2+q^2x^2}$
2. $\int_{-\infty}^{\infty} e^{-x}$
3. $\int_{-\infty}^{\infty}\frac{dx}{1+x^2}$
4. $\int_{-\infty}^{1}\frac{dx}{1+x^2}$

70. If $A=\begin{bmatrix}1&0&0\\1&0&1\\0&1&0\end{bmatrix}$ then which of the following is/are true :

1. $A^5=A^3+A^2-I$
2. $A^6=A^4+A^2-I$
3. $A^7=A^5+A^2-I$
4. $A^8=A^6+A^2-I$

71. Let $A$ is a $4\times4$ matrix with integer entries. Suppose the system $Ax=b$ has a non-real solution for some $4\times1$ integer matrix $b$. Then

1. $Ax=b$ has an integer solution
2. $Ax=b$ has an rational solution
3. $\exists\, b\in\mathbb{R}^n$ such that $Ax=b$ is inconsistent
4. All of above

72. Which of the following is/are subspace of $\mathbb{R}^3$ ?

1. $\{(3t,2t+1,5t);t\in\mathbb{R}\}$
2. $\{(x,y,z)\in\mathbb{R} : x+y+z=0\}$
3. $\{(x,y,z)\in\mathbb{R} : x^4+y^4+z^4=0\}$
4. $\{(x,y,z)\in\mathbb{R} : y=0\}$

73. Let $\{v_1, v_2, v_3\}$ be a basis of a vector space $V$ then which of the following is/are also a basis of $V$

1. $\{v_1, v_1+v_2, v_1+v_2+v_3\}$
2. $\{v_1+v_2+v_3,\ v_1-v_2-v_3\}$
3. $\{v_1+v_2+v_3, 2v_1+3v_2+4v_3, 4v_1+5v_2+6v_3\}$
4. All of the above

74. Let $a,b,c,d \in \mathbb{R}$ and let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix} \forall \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2$. Let $S:\mathbb{C} \to \mathbb{C}$ be the corresponding map defined by $S(x+iy) = (ax+by)+i(cx+dy)$ for $x, y \in \mathbb{R}$ then

1. $S$ is always $\mathbb{C}$-linear i.e., $S(z_1+z_2) = S(z_1)+S(z_2)$ for all $z_1, z_2 \in \mathbb{C}$ and $S(\alpha z) = \alpha S(z)$ for all $\alpha \in \mathbb{C}, z \in \mathbb{C}$
2. $S$ is $\mathbb{C}$-linear if $b=-c$ and $d=a$
3. $S$ is $\mathbb{C}$ linear only if $b=-c$ and $d=a$
4. $S$ is $\mathbb{C}$ linear iff $T$ is the identity transformation

75. Which of the following is/are true :

1. There exists two $n \times n$ real matrices $A$ and $B$ such that $AB - BA = I_n$
2. There exists two $n \times n$ matrices $A$ and $B$ over the field $\mathbb{Z}_2$ such that $AB - BA = I_n$
3. Both are true
4. Both are false

76. Let $A$ be a $4 \times 4$ real nilpotent matrix with index of nilpotency 4 then which of the following is/are true :

1. $I+A$ is invertible
2. $I+A$ is invertible and $(I+A)^{-1} = I - A + A^2 - A^3$
3. $I+A$ is not invertible
4. $I+A$ is diagonalizable

77. Which of the following quadratic forms is/are positive definite ?

1. $6x^2+3y^2+3z^2-4xy-2yz+4zx$
2. $2x_1^2+4x_2^2+8x_3^2-2x_2x_3-2x_1x_3+2x_1x_2$
3. $-3x^2-4y^2+2z^2-2yz+xz+7xy$
4. All of the above

78. The matrix $\begin{bmatrix} 1&1&1&1 \\ 1&1&1&1 \\ 1&1&1&1 \\ 1&1&1&1 \end{bmatrix}$ is similar to which of the following matrices :

1. $\begin{bmatrix} 4&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix}$
2. $\begin{bmatrix} 2&0&0&0 \\ 0&2&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix}$
3. $\begin{bmatrix} 2&0&0&0 \\ 0&0&0&0 \\ 0&0&2&0 \\ 0&0&0&0 \end{bmatrix}$
4. $\begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&2&0 \\ 0&0&0&2 \end{bmatrix}$

## Unit – II

79. Let $R_1 = \mathbb{Z}[x]/\langle x^3+x^2+x+1 \rangle$ and $R_2 = \mathbb{Z}[x]/\langle x^2-x-1 \rangle$, then which of the following is/are true :

1. $R_1$ is not an integral domain
2. $R_1$ is not a field
3. $R_2$ is not an integral domain
4. $R_2$ is not a field

80. Consider the Mobius transformation defined by $f(z) = \dfrac{iz+2}{4z+i}$ then which of the following is/are not true :

1. The image of real axis is a circle of area $\frac{\pi}{4}$
2. The image of real axis is a circle of area $\frac{\pi}{8}$
3. The image of imaginary axis is a circle

of area $\frac{\pi}{4}$.

4. The image of imaginary axis is a circle of area $\frac{\pi}{8}$.

81. Let $K \subseteq \mathbb{C}$ be a bounded set. Let $H(\mathbb{C})$ denote the set of all entire functions and let $C(K)$ denote the set of all continuous functions on K. Consider the restriction map $r: H(\mathbb{C}) \to C(K)$ given by $r(f) = f_K$. Then $r$ is injective if
1. K is compact.
2. K is connected.
3. K is uncountable.
4. K is finite.

82. For $z \in \mathbb{C}$, define $f(z) = \frac{e^{iz}+1}{e^z}$. Then
1. $f$ is entire.
2. the only singularities of $f$ are poles.
3. $f$ has infinitely many poles on the imaginary axis.
4. $z = \infty$ is an isolated essential singularity of $f$.

83. Let $D = \{z \in \mathbb{C} : |z| < 1\}$. Then there exists a holomorphic function $f : D \to \bar{D}$ with $f(0) = 0$ with the property
1. $f'(0) = \frac{1}{7}$
2. $\left|f\left(\frac{1}{3}\right)\right| = \frac{1}{5}$
3. $f\left(\frac{1}{7}\right) = \frac{1}{5}$
4. $|f'(0)| = \operatorname{cosec}\left(\frac{\pi}{6}\right)$

84. Let $f(x) = x^4 + 3x^3 - 9x^2 + 7x + 27$ and let $p$ be a prime. Let $f_p(x)$ denote the corresponding polynomial with coefficients in $\mathbb{Z}/p\mathbb{Z}$. Then
1. $f_2(x)$ is irreducible over $\mathbb{Z}/2\mathbb{Z}$.
2. $f(x)$ is irreducible over $\mathbb{Q}$.
3. $f_3(x)$ is irreducible over $\mathbb{Z}/3\mathbb{Z}$.
4. $f(x)$ is irreducible over $\mathbb{Z}$.

85. Which of the following symmetric groups contain an element of order 30
1. $S_{10}$
2. $S_9$
3. $S_{11}$
4. $S_{12}$

86. Consider the multiplicative group $G$ of all the (complex) $2^n$-th roots of unity where $n = 0, 1, 2$ ... Then
1. Every proper subgroup of $G$ is finite.
2. $G$ has a finite set of generators.
3. $G$ is cyclic.
4. Every finite subgroup of $G$ is cyclic.

87. Let $m = \left[\mathbb{Q}\left(e^{\frac{\pi i}{40}}\right) : \mathbb{Q}\right]$ and $n = \left[\mathbb{Q}\left(e^{\frac{\pi i}{20}}\right) : \mathbb{Q}\right]$, then which of the following is/are true :
1. $m$ divides $n$
2. $n$ divides $m$
3. $m \le n$
4. $n \le m$

88. We are given a class consisting of 4 boys and 4 girls. A committee that consists of a President, a Vice-President and a Secretary is to be chosen among the 8 students of the class. Let $a$ denote the number of ways of choosing the committee in such a way that the committee has at least one boy and at least one girl. Let $b$ denote the number of ways choosing the committee in such a way that the number of girls is greater than or equal to that of the boys.
Then
1. $a = 288$
2. $b = 168$
3. $a = 144$
4. $b = 192$

89. Pick the correct statements :
1. $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(i)$ are isomorphic as $\mathbb{Q}-$ vector spaces.
2. $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(i)$ are isomorphic as fields.
3. $Gal_{\mathbb{Q}}(\mathbb{Q}(\sqrt{2})/\mathbb{Q}) \cong Gal_{\mathbb{Q}}(\mathbb{Q}(i)/\mathbb{Q})$
4. $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(i)$ are both Galois

extensions of $\mathbb{Q}$.

90. Consider the linear congruences $x \equiv 1(\text{mod}\,3), x \equiv 2(\text{mod}\,4), x \equiv 3(\text{mod}\,5)$, then which of the following intervals contain exactly one common solution of these congruences
   1. $[0,100]$  2. $[100,200]$
   3. $[200,300]$  4. $[300,400]$

**Unit - III**

91. Let $K$ be a real constant. The solution of the differential equation $\frac{dy}{dx} = 2y + z$ and $\frac{dz}{dx} = 3y$ satisfies the relation
   1. $y - z = Ke^{3x}$  2. $3y + z = Ke^{3x}$
   3. $3y - z = Ke^{3x}$  4. $y + z = Ke^{3x}$

92. Which of the following statements are true :
   1. The eigen values of Sturm-Liouville problem are all real
   2. For Sturm-Liouville problem $\exists$ an infinite number of characteristic values ( eigen values )
   3. For each eigen value of Sturm-Liouville problem there exists one and only one linearly independent eigen function
   4. Out of three statements two statements are true

93. The given differential $\frac{d}{dx}\left(x\frac{dy}{dx}\right) + \frac{\lambda}{x}y = 0$, $(\lambda > 0)$ satisfying the boundary conditions $y'(1) = 0$ and $y'(e^{\pi}) = 0$ which is/are true :
   1. $\lambda_n = n^2$
   2. $\phi_n(x) = a_n \sin(n \log x)$, $n = 1, 2, \ldots$ and $1 < x < e^{\pi}$
   3. $\lambda_n = (2n+1)^2$
   4. $\phi_n(x) = a_n \cos((2n+1)\log x)$, $n = 1, 2, \ldots$  $1 < x < e^{\pi}$

94. For the given PDE $uu_x + yu_y = x$ which is/are not true :
   1. $f\left(u^2 - x^2, \frac{y}{x+u}\right) = 0$, where $f : \mathbb{R}^2 \to \mathbb{R}^2$ is $c'$ and $\nabla f \neq (0,0)$ at every point
   2. $u^2 = g\left(\frac{y}{x+u}\right) + x^2$, $g \in c'(\mathbb{R})$
   3. $f\left(u^2 + x^2\right) = 0$, $f \in c'(\mathbb{R})$
   4. $f(x+y) = 0$, $f \in c'(\mathbb{R})$

95. Let $u = u(x,y)$ be the complete integral of the PDE $pq = xy$ passing through $(0,0,1)$ and $\left(0,1,\frac{1}{2}\right)$ in the $x - y - z$ space. Then the value of the $u(x,y)$ evaluated at $\left(-\sqrt{2}, \sqrt{2}\right)$ is/are
   1. 0  2. 1
   3. $-1$  4. None

96. The functional $J(y) = \int_0^1 \left(y'^2 + x^2\right)dx$ where $y(0) = -1$ and $y(1) = 1$
   1. $y = 2x^2 - 1$ is extremal which is weak minimum
   2. $y = 2x - 1$ is extremal which is weak minimum
   3. $y = 2x^2 - 1$ is extremal which is strong minimum
   4. $y = 2x - 1$ is extremal which is strong minimum

97. Shortest distance between the point $A(-1,3)$ and the straight line $y = 1 - 3x$ is/are
   1. $\frac{1}{\sqrt{10}}$  2. $\frac{1}{\sqrt{20}}$
   3. $\frac{\sqrt{10}}{\sqrt{20}}$  4. None

98. PDE of the given equation $z = f(xy) + \phi\left(\frac{x}{y}\right)$ is/are
   1. $x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0$
   2. $x^2\frac{\partial^2 z}{\partial x^2} + y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$
   3. $y^2\frac{\partial^2 z}{\partial y^2} - x^2\frac{\partial^2 z}{\partial x} - x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$
   4. None

99. Corresponding Volterra integral equation is/are $y'''-2xy=0,\ y(0)=\frac{1}{2},$ $y'(0)=y''(0)=1$

1. $y(x)=\frac{1}{2}+x+\frac{x^2}{2}+\int_0^x \xi(x-\xi)^2 y(\xi)d\xi$
2. $y(x)=\frac{1}{2}+\frac{x^2}{2}-x+\int_0^x \xi(x-\xi)^2 y(\xi)d\xi$
3. $u(x)=x(x+1)^2+\int_0^x x(x-t)^2 u(t)dt$
4. $u(x)=x(x-1)^2+\int_0^x x(x-t)^2 u(t)dt$

100. For the integral equation

$$\phi(x)=f(x)+\lambda\int_0^1(1-3xt)\phi(t)dt$$

1. If $f(x)=0$ and $\lambda=2$, then it has infinite solutions.
2. If $f(x)=0$ and $\lambda=-2$, then it has infinite solutions.
3. If $f(x)\neq 0$ and $\lambda\neq\pm 2$, then it has unique solution.
4. If $f(x)=0$ and $\lambda\neq\pm 2$ , then it has unique solution.

101. Consider a particle of mass $m$ in simple harmonic oscillation about the origin with spring constant $k$ ; then for the Lagrangian L and the Hamiltonian H of the system

1. $L(x,\dot{x})=\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2$

   $H(x,\dot{x})=\frac{p^2}{2m}+\frac{1}{2}kx^2$ ; $p$ is generalized momentum
2. $L(x,\dot{x})=\frac{1}{2}m\dot{x}^2+\frac{1}{2}kx^2$ and the generalized momentum is $p=m\dot{x}$.
3. $L(x,\dot{x})=\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2$ and the generalized momentum is $p=m\dot{x}$.
4. $L(x,\dot{x})=\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2$

   $H(x,\dot{x})=\frac{m\dot{x}^2}{2}+\frac{1}{2}kx^2$

102. Consider the function $f(x)=\sqrt{2+x}$ for $x\geq -2$ and the iteration $x_{n+1}=f(x_n);n\geq 0$ for $x_0=1$. What are the possible limits of the iteration ?

1. $\sqrt{2+\sqrt{2+\sqrt{2+...}}}$
2. $-1$
3. 2
4. 1

**Unit - IV**

103.Consider the following primal Linear Programming Problem.

max $z=-3x_1+2x_2$

subject to

$x_1\leq 3$

$x_1-x_2\leq 0$

$x_1,x_2\geq 0$

Which of the following statements are true ?

1. The primal problem has an optimal solution.
2. The primal problem has an unbounded solution.
3. The dual problem has an unbounded solution.
4. The dual problem has no feasible solution.

104.Suppose that system 1 has 2 components $C_1$ and $C_2$ in series while system 2 has 2 components $C_3$ and $C_4$ in parallel. The components $C_1,C_2,C_3$ and $C_4$ have independent and identically distributed life times each being exponential with mean 1. Suppose $S_i(t)$ and $h_i(t)$ are the survival and hazard rate function, respectively, for the $i$-th system, $i=1,2$. Then which of the following statements are true ?

1. $S_1(t) < S_2(t)$ for all $t > 0$.
2. $h_1(t) < h_2(t)$ for all $t > 0$.
3. The expected life time of the system 1 is $\frac{1}{2}$.
4. The expected life time of the system 2 is 1.

105.Consider an experiment using a balanced incomplete block design with $v = 4$ treatments, $b = 6$ blocks and block size $k = 2$. Let $t_i\,(i=1,2,3,4)$ be the effect of the *i*-th treatment and $\sigma^2$ be the variance of an observation. Which of the following statements are true ?

1. The variance of the best linear unbiased estimator (BLUE) of $\sum_{i=1}^{4} p_i t_i$ where $\sum_{i=1}^{4} p_i = 0$ and $\sum_{i=1}^{4} p_i^2 = 1$ is $\sigma^2 / 2$.
2. The covariance between the BLUEs of the contrasts $\sum_{i=1}^{4} p_i t_i$ and $\sum_{i=1}^{4} q_i t_i$ where $\sum_{i=1}^{4} p_i q_i = 0$ is zero.
3. The degrees of freedom associated with the error sum of squares is 3.
4. The efficiency factor of the design relative to a randomized block design with 3 replicates is 2/3.

106.Consider a finite population containing $N = nk$ units, $n \geq 2, k \geq 2$ being integers and let these units be numbered 1 to N is some order. In order to select a sample of *n* units, a unit is selected at random from the first *k* units, and every *k*-th unit thereafter. Under this scheme, let $\pi_i$ be the probability that the *i*-th unit is included in the sample and $\pi_{ij}$ be the probability that both *i*-th and *j*-th units are included in the sample. Also, let $\bar{y}$ denote the sample mean of a study variable, say *y*. Which of the following statements are true ?

1. $\pi_i = \frac{n}{N}, \pi_{ij} = \frac{n(n-1)}{N(N-1)}$ for all $i, j = 1, 2, ...., N, i \neq j$.
2. $\pi_i = \frac{n}{N}$, for all $i = 1, 2, ...., N$ and $\pi_{ij} = 0$ for at least one pair $(i, j)\, i, j = 1, 2, ...., N$, $i \neq j$.
3. $\pi_i = \frac{1}{N}$, for all $i, j = 1, 2, ...., N$ and $\pi_{ij} > 0$ for all $i \neq j$, $i, j = 1, 2, ...., N$.
4. $N\,\bar{y}$ is an unbiased estimator of the population total.

107.Aerial observations $Y_1, Y_2, Y_3$ and $Y_4$ are made on angles $\theta_1, \theta_2, \theta_3$ and $\theta_4$ respectively, of a quadrilateral on the ground. If the observations $\{Y_i, i = 1, 2, 3, 4\}$ are subject to normal errors with mean 0 and variance $\sigma^2$, then which of the following statements are true ?

1. The best linear unbiased estimator of $\theta_i$ is $\hat{\theta}_i = Y_i - \bar{Y} + \frac{\pi}{2}$, $i = 1, 2, 3, 4$, where $\bar{Y} = \frac{1}{4}\sum_{i=1}^{4} Y_i$.
2. The best linear unbiased estimator of $\theta_i$ is $\hat{\theta}_i = Y_i$, $i = 1, 2, 3, 4$.
3. The error sum of squares is $4\left(\bar{Y} - \frac{\pi}{2}\right)^2$.
4. The error sum of squares is $\sum_{i=1}^{4}\left(Y_i - \frac{\pi}{2}\right)^2$

108.For any set of data which of the following statements are NOT possible ? (Notations have their usual significance)

1. $r_{1.234} = 0.47, r_{1.23} = 0.52$
2. $r_{1.23} = -0.32, r_{12.3} = -0.23$
3. $r_{12} = 0.3, r_{13} = 0.2, r_{12.3} = -0.23$
4. $r_{1.234} = 0.47, r_{12} = 0.73$

109.Let $X_1, X_2, ...., X_n$ be a random sample from $f_\theta(x) = \begin{cases} \theta e^{-\theta x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$

Consider the problem of testing $H_0: \theta = 1$ against $H_1: \theta > 1$

Define

$$\varphi_1 = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} x_i > c_1 \\ 0 & \text{if } \sum_{i=1}^{n} x_i < c_1 \end{cases} \quad \text{and}$$

$$\varphi_2 = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} x_i < c_2 \\ 0 & \text{if } \sum_{i=1}^{n} x_i \ge c_2 \end{cases}$$

where $c_1$ and $c_2$ are such that $\varphi_1$ and $\varphi_2$ are of size $\alpha$. Which of the following statements are true ?

1. $\varphi_1$ is more powerful than $\varphi_2$.
2. P-value of the uniformly most powerful test for testing $H_0$ against $H_1$ is given by $P_{\theta_0}\left[\sum_{i=1}^{n} X_i > \text{observed } \sum_{i=1}^{n} x_i\right]$
3. Power function of $\varphi_2$ is monotonically increasing.
4. $\varphi_1$ is unbiased.

110.Let $X_1, X_2, ...., X_n$ be independent and identically distributed random variables with common continuous distribution function F, which is symmetric about the median $\mu$. Consider the problem of testing $H_0 : \mu = 0$ against $H_1 : \mu > 0$. Define

$R_i^+ =$ Rank of $|X_i|$ among $|X_1|, |X_2|, ...., |X_n|, i = 1, 2, ..., n.$

$L =$ Sum of the $R_i^+$'s, for which $X_i < 0$, $i = 1, 2, ...., n$.

$G =$ Sum of $R_i^+$'s, for which $X_i > 0$, $i = 1, 2, ...., n$.

Which of the following statements are true ?

1. Left tailed test based on L is appropriate for testing $H_0$ against $H_1$.
2. Right tailed test based on G is appropriate for testing $H_0$ against $H_1$.
3. Maximum possible value of L is $n(n+1)$.
4. $E_{H_1}(L+G) = \frac{n(n+1)}{2}$

111.Let $X_1, X_2, ...., X_n$ be a random sample from $f_\theta(x) = \begin{cases} e^{-(x-\theta)}, & x > \theta \\ 0, & x \le \theta \end{cases}$

Define $X_{(1)} = \min\{X_1, X_2, ...., X_n\}$. Which of the following are confidence intervals for $\theta$ with confidence coefficient $(1-\alpha)$ ?

1. $\left[X_{(1)} + \frac{1}{n}\log_e \alpha, X_{(1)}\right]$.
2. $\left[X_{(1)} + \frac{1}{n}\log_e \alpha, X_{(1)} - \frac{1}{n}\log_e \alpha\right]$.
3. $\left[X_{(1)} + \frac{1}{n}\log_e\left(\frac{\alpha}{2}\right), X_{(1)} + \frac{1}{n}\log_e\left(1-\frac{\alpha}{2}\right)\right]$
4. $\left[X_{(1)} + \frac{1}{n}\log_e \alpha, X_{(1)} - \frac{1}{n}\log_e\left(1-\frac{\alpha}{2}\right)\right]$

112.Suppose $X_1, X_2, ...., X_n$ are independent and identically distributed as geometric random variables with parameter *p*. Let *f* denote the number $X_i$'s equal to 1. Then which of the following statements are true ?

1. $\frac{f}{n}$ is the maximum likelihood estimate of *p*.
2. $\frac{f}{n}$ is an unbiased estimator of *p*.
3. $\frac{n-1}{\sum_{i=1}^{n} x_i}$ is the maximum likelihood estimator of *p*.
4. $Var\left(\frac{f}{n}\right) = \frac{p(1-p)}{n}$

113.Consider the following random sample of size 11 from uniform $(\theta-1, \theta+1)$ distribution :

–0.71, 0.3, –0.4, –0.63, –0.81, –0.7, 0.1, –0.01, 0.02, –0.96, –0.92.

Which of the following are maximum likelihood estimates of $\theta$ ?

1. –0.96 2. 0.3

3. 0.02    4. –0.54

114. Let X and Y be two independent $N(0,1)$ random variables. Define $U = \frac{x}{y}$ and $V = \frac{X}{|Y|}$. Then
1. U and V have the same distribution.
2. V has *t* distribution.
3. $E\left(\frac{V}{U}\right) = 0$.
4. U and V are independent.

115. Let X and Y be independent and identically distributed random variables having a normal distribution with mean 0 and variance 1. Define Z and W as follows

$$\begin{pmatrix} Z \\ W \end{pmatrix} = \begin{cases} \begin{pmatrix} X \\ Y \end{pmatrix} & \text{if} \quad XY > 0 \\ \begin{pmatrix} -X \\ Y \end{pmatrix} & \text{if} \quad X < 0 \text{ and } Y > 0 \\ \begin{pmatrix} X \\ -Y \end{pmatrix} & \text{if} \quad X > 0 \text{ and } Y < 0 \end{cases}$$

Then
1. Z and W are independent.
2. Z has N(0, 1) distribution.
3. W has N(0, 1) distribution.
4. $Cov(Z, W) > 0$.

116. A fair coin is tossed repeatedly. Let X be the number of Tails before the first Head occurs. After the first Head occurred, an additional Y Tails appear before the next Head occurs. Which of the following statements are true ?
1. P(X is even, Y is even) = P(X is odd, Y is odd).
2. P(X is even, Y is even) = P(X is even, Y is odd).
3. P(X is even, Y is even) > P(X is even, Y is odd).
4. P(X is even, Y is even) < P(X is even, Y is odd).

117. Let $X_n$ be distributed as a Poisson random variable with parameter *n*. Then which of the following statements are correct ?
1. $\lim_{n\to\infty} P\left(X_n > n + \sqrt{n}\right) = 0$
2. $\lim_{n\to\infty} P\left(X_n \le n + \sqrt{n}\right) = 0$
3. $\lim_{n\to\infty} P\left(X_n \le n\right) = \frac{1}{2}$
4. $\lim_{n\to\infty} P\left(X_n \le n\right) = 1$

118. Let $(X_n)_{n\ge 0}$ be Markov chain on state space $S = \{-N, -N+1, ...., -1, 0, 1, ..., N-1, N\}$ with the transition probabilities given by

$p_{i,i+1} = p_{i,i-1} = \frac{1}{2}$ for all $-N+1 \le i \le N-1$

$p_{N,N-1} = p_{-N,-N+1} = p_{N,N} = p_{-N,-N} = \frac{1}{2}$.

Then
1. $(X_n)_{n\ge 0}$ has a unique stationary distribution.
2. $(X_n)_{n\ge 0}$ is irreducible.
3. $\lim_{n\to\infty} P\left(X_n = N \mid X_0 = 0\right) = \lim_{n\to\infty} P\left(X_n = -N \mid X_0 = 0\right)$
4. $(X_n)_{n\ge 0}$ is recurrent.

119. Suppose U and V are independent and identically distributed random variables with $P(U = i) = P(V = i) = \frac{1}{4}$ for $i = 1, 2, 3, 4$. Consider the triangle T, bounded by the $x-$axis, $y-$axis and the line $Ux + Vy = UV$. Then which of the following statements are true ?
1. $P\left(Area(T) < 2\right) = \frac{5}{16}$
2. $P\left(T \text{ is isosceles}\right) = \frac{1}{4}$
3. $P\left(Area(T) \le 8\right) = 1$
4. $P\left(Area(T) > 1\right) = 1$

120. For any set of data, which of the following statements are true ?
1. Standard deviation $\le \frac{1}{2}(\text{range})$.
2. Mean absolute deviation about mean $\le$ standard deviation.
3. Mean absolute deviation about median $\le$ standard deviation.
4. Mean absolute deviation about mode $\le \frac{1}{2}$ (range).

## Answer Key

| Part - A | | Part - B | | | | Part - C | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | 4 | 21. | 1 | 51. | 4 | 61. | 1,2,3,4 | 91. | 2 |
| 2. | 3 | 22. | 4 | 52. | 2 | 62. | 1,2 | 92. | 1,2,4 |
| 3. | 3 | 23. | 3 | 53. | 3 | 63. | 1,4 | 93. | 1,2 |
| 4. | 2 | 24. | 3 | 54. | 3 | 64. | 1,4 | 94. | 3,4 |
| 5. | 1 | 25. | 3 | 55. | 3 | 65. | 1,2 | 95. | 3 |
| 6. | 1 | 26. | 2 | 56. | 1 | 66. | 1,2 | 96. | 2,4 |
| 7. | 1 | 27. | 2 | 57. | 3 | 67. | 2,3 | 97. | 1 |
| 8. | 1 | 28. | 4 | 58. | 2 | 68. | 2,3,4 | 98. | 1,3 |
| 9. | 3 | 29. | 3 | 59. | 3 | 69. | 1,3,4 | 99. | 1,3 |
| 10. | 4 | 30. | 3 | 60. | 3 | 70. | 1,2,3,4 | 100. | 1,2,3,4 |
| 11. | 1 | 31. | 2 | | | 71. | 2,3 | 101. | 1,3,4 |
| 12. | 2 | 32. | 1 | | | 72. | 2,3,4 | 102. | 1,3 |
| 13. | 3 | 33. | 2 | | | 73. | 1 | 103. | 2,4 |
| 14. | 2 | 34. | 2 | | | 74. | 2,3 | 104. | 1,3 |
| 15. | 2 | 35. | 4 | | | 75. | 2 | 105. | 1,2,3,4 |
| 16. | 4 | 36. | 4 | | | 76. | 1,2 | 106. | 2,4 |
| 17. | 2 | 37. | 2 | | | 77. | 1,2 | 107. | 1,3 |
| 18. | 3 | 38. | 2 | | | 78. | 1 | 108. | 1,2,3,4 |
| 19. | 2 | 39. | 4 | | | 79. | 1,2,4 | 109. | 3 |
| 20. | 4 | 40. | 3 | | | 80. | 1,2,3, | 110. | 1,2,4 |
| | | 41. | 4 | | | 81. | 3 | 111. | 1,2,3,4 |
| | | 42. | 1 | | | 82. | 1,4 | 112. | 2,4 |
| | | 43. | 2 | | | 83. | 1,2 | 113. | 3,4 |
| | | 44. | 3 | | | 84. | 1,2,4 | 114. | 1,2,3 |
| | | 45. | 1 | | | 85. | 1,3,4 | 115. | 2,3,4 |
| | | 46. | 2 | | | 86. | 1,4 | 116. | 3 |
| | | 47. | 3 | | | 87. | 3,4 | 117. | 3 |
| | | 48. | 3 | | | 88. | 1,2 | 118. | 1,2,3,4 |
| | | 49. | 4 | | | 89. | 1,3,4 | 119. | 1,2,3 |
| | | 50. | 4 | | | 90. | 1,4 | 120. | 1,2,3 |