INSTRUCTIONS

1. You have opted for English as medium of question paper. This test Booklet contains one hundred and twenty (20 Part' A' +40 Part 'B' +60 Part 'C') Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 25 and 20 questions from part 'A' 'B' and 'C' respectively. If more than required number of questions are answered, only first 15, 25 and 20 questions in Parts 'A' 'B' and 'C' respectively, will be taken up for evaluation.
2. Each question in Part 'A' carries 2 marks, Part 'B' 3 marks and Part 'C' 4.75 marks respectively. There will be negative marking @0.5 marks in Part 'A' and @ 0.75 marks in Part 'B' for each wrong answer and no negative marking for Part 'C'.
3. Below each question in Part 'A' and 'B', four alternatives or responses are given. Only one of these alternatives is the "correct" option to the question. You have to find, for each question, the correct or the best answer. In Part 'C' each question may have "**ONE**' or '**MORE**' correct options in Part 'C'. Credit in a question shall be given only on idenfication of 'ALL' the correct options in Part 'C'. No credit shall be allowed in a question if any incorrect option is marked as correct answer.

Part - A

1. $\dfrac{128352}{238368}$ when reduced to its lowest terms is :

   1. $\dfrac{3}{4}$ 2. $\dfrac{7}{13}$
   3. $\dfrac{9}{13}$ 4. $\dfrac{5}{13}$

2. The mean of five numbers is 18. If one number is excluded, their mean is16. The excluded number is :

   1. 25 2. 26
   3. 27 4. 30

3. The mean of 100 items was found to be 30. If at the time of calculation, two items were wrongly taken as 32 and 12 instead of 23 and 11, the correct mean is :

   1. 29.4 2. 29.5
   3. 29.8 4. 29.9

4. 16% of a number is 216. What is 27% of that number ?

   1. 274 2. 367.20
   3. 279 4. 364.50

5. A man sold and article at a loss of 20%. If he sells the article for Rs. 12 more, he would have gained 10%. The cost price of the article is :

   1. Rs. 60 2. Rs. 40
   3. Rs. 30 4. Rs. 22

6. A man sold two pipes at Rs. 12 each. On one he gained 20% and on the other lost 20%. On the whole, he :
   1. neither gained nor lost
   2. gained Rs. 1
   3. lost Rs. 1
   4. gained Rs. 2
7. A does 20% less work than *B*. If *A* can complete a piece of work in $7\frac{1}{2}$ hours, then *B* can do it in
   1. 5 hours 2. $5\frac{1}{2}$ hours
   3. 6 hours 4. $6\frac{1}{2}$ hours
8. In what time can Sonali cover a distance of 400m, if she runs at a speed of 20 km/hr ?
   1. $1\frac{1}{5}$ minutes 2. $1\frac{1}{2}$ minutes
   3. 2 minutes 4. 3 minutes
9. A man covers half of his journey at 6 km/hr and the remaining half at 3 km/hr. His average speed is :
   1. 3 km/hr 2. 4 km/hr
   3. 4.5 km/hr 4. 9 km/hr
10. A 120 m long train takes 10 seconds to cross a man standing on a platform. The speed of the train is :
   1. 10 m/sec 2. 12 m/sec
   3. 15 m/sec 4. 20 m/sec
11. A boat is rowed downstream at 15.5 km/hr and upstream at 8.5 km/hr. The speed of the stream is :
   1. 3.5 km/hr 2. 5.75 km/hr
   3. 6.5 km/hr 4. 7 km/hr
12. A sum of money amounts to Rs 5200 in 5 years and to Rs 5680 in 7 years at simple interest. The rate of interest per annum is :
   1. 3% 2. 4%
   3. 5% 4. 6%
13. Roshan invested Rs 10000 at compound interest at 10% p.a. for a period of 3 years. What amount will he get after 3 years ?
   1. Rs 12310 2. Rs 13120
   3. Rs 13320 4. None of these
14. The whole surface area of a cuboid 24 cm long, 14 cm broad and 7.5 cm high, is :
   1. $2520\ cm^2$ 2. $1260\ cm^2$
   3. $1242\ cm^2$ 4. $621\ cm^2$
15. The length of the diagonal of a cuboid 30 cm long, 24 cm broad and 18 cm high, is :
   1. 30 cm 2. $15\sqrt{2}$ cm
   3. $30\sqrt{2}$ cm 4. 60 cm
16. January 1, 2007 was Monday. What day of the week lies on Jan 1, 2008 ?
   1. Monday 2. Tuesday
   3. Wednesday 4. Sunday
17. A clock is started at noon. By 10 minutes past 5, the hour hand has turned through :
   1. 145° 2. 150°
   3. 155° 4. 160°

18. An angle which is greater than 180° but less than 360° is called
    1. an acute angle 2. an obtuse angle
    3. a straight angle 4. a reflex angle

19. 1, 9, 25, 49, ?, 121
    1. 64 2. 81
    3. 91 4. 100

20. 4, 7, 12, 19, 28, ?
    1. 30 2. 36
    3. 39 4. 49

Part - B

Unit – 1

21. Suppose $\langle a_n \rangle$ is a sequence of real numbers such that $a_n \to l \neq 0$ then $\lim_{n\to\infty}\left(\frac{a_n - l}{a_n + l}\right)$ is
    1. 0
    2. 1
    3. l
    4. Cannot be determined

22. Let $\langle a_n \rangle$ be a divergent sequence of real numbers such that $a_n > 0$ for all n and let $b_n = (a_1 \cdot a_2 \cdots a_n)^{\frac{1}{n}}$ then $\lim_{n\to\infty} b_n$ is
    1. 0 2. 1
    3. ∞ 4. None of these

23. Which of the following is best on the open interval $\left(0, \frac{\pi}{2}\right)$
    1. $x > \tan x > \sin x$
    2. $x \geq \tan x \geq \sin x$
    3. $\tan x \geq x \geq \sin x$
    4. $\tan x > x > \sin x$

24. Let $a_n \geq 0 \ \forall \ n$ and the series $\sum_{n=1}^{\infty} a_n$ is convergent then the series $\sum_{n=1}^{\infty} \frac{\sqrt{a_n}}{n}$
    1. may or may not convergent
    2. must be convergent
    3. must be divergent
    4. may oscillate

25. Which of the following intervals is best for the validity of the expansion
    $$\tan^{-1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ...$$
    1. $(-1,1)$ 2. $(-1,1]$
    3. $[-1,1)$ 4. $[-1,1]$

26. Let $\langle a_n \rangle$ be a monotonic sequence of real numbers and it has a convergent subsequence that converges to $l > 0$ then which of the following is true :
    1. $\langle a_n \rangle$ may not be convergent.
    2. $\langle a_n \rangle$ is convergent but it is not necessary that it converges to l
    3. $\langle a_n \rangle$ is convergent only if $a_n > 0 \ \forall \ n$

4. $\langle a_n \rangle$ converges to l

27. Let A and B be square complex matrices such that A commutes with B and B^* then which of the following is true :

1. $AA^* = A^*A$
2. $BB^* = B^*B$
3. $\left(A+A^*\right)\left(B+B^*\right) = \left(B+B^*\right)\left(A+A^*\right)$
4. None of these

28. If $A = \begin{pmatrix} -15 & 12 \\ -24 & 19 \end{pmatrix}$ then e^{At} is

1. $\begin{pmatrix} 9e^t - 8e^{3t} & 6e^{3t} - 6e^t \\ 12e^t - 12e^{3t} & 9e^{3t} - 8e^t \end{pmatrix}$
2. $\begin{pmatrix} 9e^t + 8e^{3t} & 6e^{3t} + 6e^t \\ 12e^t - 12e^{3t} & 9e^{3t} - 8e^t \end{pmatrix}$
3. $\begin{pmatrix} 9e^t + 8e^{3t} & 6e^{3t} - 6e^t \\ 12e^t + 12e^{3t} & 9e^{3t} + 8e^t \end{pmatrix}$
4. None of the above

29. Let A and B are n-square upper triangular matrices of rank $n-1$ then

1. $\text{rank}(AB) = n-1$
2. $\text{rank}(AB) = n-2$
3. $\text{rank}(AB) = 0$
4. None of the above

30. Let V be the subspace of $\mathbb{R}^4$ spanned by $\alpha_1 = (1,2,3,4)$, $\alpha_2 = (2,3,4,5)$, $\alpha_3 = (3,4,5,6)$, $\alpha_4 = (4,5,6,7)$ then $\dim(V)$ is :

1. 3
2. 4
3. 1
4. 2

31. Let $A = \begin{pmatrix} 2 & 3 & 2 \\ 1 & 4 & 2 \\ 1 & -3 & 1 \end{pmatrix}$ then eigen values of A are :

1. 4, 2, 1
2. 3, 3, 1
3. 2, 2, 3
4. None

32. The value of $\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!}$ is :

1. $\frac{e}{2}$
2. $\frac{2}{e}$
3. $\frac{e}{2} + \frac{2}{e}$
4. None

Unit - II

33. Let G be a group (finite or infinite) such that G has more than 16 elements of order 17 then

1. G must be abelian
2. G must be cyclic
3. G can not be abelian
4. G can not be cyclic

34. Let J be an ideal of the ring $M_2(\mathbb{Z}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a,b,c,d \in \mathbb{Z} \right\}$ such that $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \in J$ then

1. J is a prime ideal
2. J is a maximal ideal
3. J is a nil ideal
4. $J = M_2(\mathbb{Z})$

35. Let $K = Q[x]/\langle x^3 - 2 \rangle$ and n denotes the number of fields lying strictly between Q and K then n is equal to

1. 1
2. 0
3. 2
4. 3

36. The radius of convergence of the power series $\sum_{n=0}^{\infty}\frac{2^n+3^n}{4^n+5^n}z^n$ is

1. 0
2. 1
3. ∞
4. None

37. Sum of coefficients of z and $\frac{1}{z}$ in the Laurent expansion of $f(z)=\frac{1}{(z-1)(z-2)}$ valid for the region $1<|z|<2$ is

1. -1
2. $-\frac{1}{2}$
3. $-\frac{3}{2}$
4. None

38. Let f and g be two entire functions such that they agree on infinitely many points then

1. $f(z)=g(z)\ \ \forall\ z\in\mathbb{C}$
2. $f(z)=g(z)\ \ \forall\ z\in\mathbb{R}$
3. $f(z)=g(z)\ \ \forall\ z$ with $|z|=1$
4. None of the above

39. Let $A\subseteq\mathbb{R}^2$ and $X=\mathbb{R}^2\setminus A$ be subsets with subspace topology inherited from the usual topology on $\mathbb{R}^2$. Then

1. A is countable dense implies that X is totally disconnected.
2. A is unbounded implies that X is compact.
3. A is open implies that X is compact.
4. A is countable implies that X is path connected.

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Unit - III

41. If $y(t)$ is a solution of the differential equation $y''+4y=2e^t$, then $\lim_{t\to\infty}e^{-t}y(t)$ is equal to

1. $\frac{2}{3}$
2. $\frac{2}{5}$
3. $\frac{2}{7}$
4. $\frac{2}{9}$

42. Let $y(x)=u(x)\sin x+v(x)\cos x$ be a solution of the differential equation $y''+y=\sec x$. Then, $u(x)$ is equal to

1. $\log|\cos x|+c$
2. $-x+c$
3. $x+c$
4. $\log|\sec x|+c$

43. Consider the equation

$$y(x)=\lambda\int_0^1 K(x,t)\,y(t)\,dt \text{ where}$$

$$K(x,t)=\begin{cases} x(1-t) & 0\le t\le 1\\ t(1-x) & 0\le t\le x\end{cases}$$

1. $\begin{cases} y''(x)+\lambda y(x)=0 \\ y(0)=y(1)=0 \end{cases}$

2. $\begin{cases} y''(x)-\lambda y(x)=0 \\ y(0)=y(1)=0 \end{cases}$

3. $\begin{cases} y''(x)+\lambda y(x)=0 \\ y(0)=y(1)\neq 0 \end{cases}$

4. None

44. If $J(y)=\int_2^3\left(y'^2+2yy'+y^2\right)dx$, $y(2)=e$ and $y(3)$ is arbitrary then extremal is

1. e^{x+1}
2. e^{-x+1}
3. e^{x-1}
4. e^{-x+3}

45. Consider the initial value problem

$\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0,\ u(0,y)=4e^{-2y}$ then the value of $u(1,2)$

1. 3
2. 2
3. 4
4. 1

46. $2\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x\partial y}+8\frac{\partial^2 z}{\partial y^2}=0$

1. Elliptic
2. Parabolic
3. Hyperbolic
4. None

47. Let $P:\mathbb{R}\to\mathbb{R}$ be a polynomial of the form $P(x)=a_0+a_1x+a_2x^2$, with $a_0,a_1,a_2\in\mathbb{R}$ and $a_2\neq 0$.

Let $E_1=\int_0^1 P(x)dx-\frac{1}{2}\left(P(0)+P(1)\right)$

$E_2=\int_0^1 P(x)dx-P\left(\frac{1}{2}\right)$

If $|x|$ is the absolute value of $x\in\mathbb{R}$, then

1. $|E_1|>|E_2|$
2. $|E_2|>|E_1|$
3. $|E_2|=|E_1|$
4. $|E_2|=2|E_1|$

48. Consider a body of unit mass falling freely from rest under gravity with velocity *v*. If the air resistance retards the acceleration by *cv* where *c* is a constant, then

1. $v=\frac{g}{c}\left[1+e^{ct}\right]$
2. $v=\frac{g}{c}\left[1+e^{-ct}\right]$
3. $v=\frac{g}{c}\left[1-e^{-ct}\right]$
4. $v=\frac{g}{c}\left[1-e^{ct}\right]$

Unit-4

49. Suppose $X_1,X_2,...,X_n$ is a random sample from $U(0,\theta),\theta>0$. Let $X_{(1)}\le X_{(2)}\le....\le X_{(n)}$ be the order statistics. Consider the two unbiased estimators for $\theta: T_1=2\bar{X}$ and $T_2=\left(\frac{n+1}{n}\right)X_{(n)}$. Then $\lim_{n\to\infty}\frac{Var(T_2)}{Var(T_1)}=$

1. 0
2. 1
3. ∞
4. 12

50. Suppose that X_1,X_2 and X_3 are independent and identically distributed random variables, each having a Bernoulli distribution with parameter $1/2$. Consider the 2×2 matrix $A=\begin{pmatrix} X_1 & 0 \\ X_2 & X_3 \end{pmatrix}$. Then,

$P(A \text{ is invertible})$ equals

1. 0
2. 1
3. 1/4
4. $3/4$

51. Consider the linear programming problem :

Minimize : $z=-2x-5y$

subject to $3x+4y\ge 5,\ x\ge 0, y\ge 0.$

which of the following is correct ?

1. Set of feasible solutions is empty.
2. Set of feasible solutions is non-empty but there is no optimal solution.
3. Optimal value is attained at $\left(0,\frac{5}{4}\right)$.
4. Optimal value is attained at $\left(\frac{5}{3},0\right)$.

52. How many distinct samples of size *n* can be drawn with replacement from the population $\{u_1,u_2,....,u_n\}$ of *n* units ?

1. n^n
2. $\binom{2n-1}{n}$
3. $\binom{2n-1}{n+1}$
4. 1

53. For testing the effectiveness of four teaching techniques, five teachers of a college were involved. A class of 120 students was divided into 5 groups of 24 each at random; one group was assigned to each of the five teachers. Each teacher further divided his/her group into four equal subgroups at random, and used one technique per subgroup. All of them used the same course material. After all the classes were over, a common examination was conducted and the marks were noted. Suppose we want to test whether all the four teaching techniques are equally effective. What is the degrees of freedom associated with the residual sum of squares?
1. 60
2. 100
3. 119
4. 460

54. Suppose $X_1, X_2, ...$ are random variables on a common probability space with $X_n \sim N(\mu_n, \sigma_n^2)$. Then, X_n converges in probability to 2 if and only if
1. $\mu_n \to 0$ and $\sigma_n^2 \to 2$
2. $\mu_n \to 2$ and $\sigma_n^2 \to 0$
3. $\mu_n \to 0$ and σ_n^2 converges
4. $\sigma_n^2 \to 0$ and μ_n converges

55. Let $X_1, X_2, ...$ be independent and identically distributed random variables with $E(X_i) = 0$ and $Var(X_i) = 1$ for all *i*. Let $S_n = X_1 + ... + X_n$. Let $\Phi(x)$ denote the cumulative distribution function of a standard normal random variable. Then, for any $x > 0$, $\lim_{n\to\infty} P(-nx < S_n < nx)$ equals
1. $2\Phi(x) - 1$
2. $\Phi(x)$
3. 1
4. $1 - \Phi(2x)$

56. Let $X_1, ..., X_n$ be a random sample of size *n* from a *p*-variate Normal distribution with mean μ and positive definite covariance matrix $\sum$. Choose the correct statement
1. $(X_1 - \mu)' \sum^{-1} (X_1 - \mu)$ has chi-square distribution with 1 d.f.
2. $\bar{X}\bar{X}'$ has Wishart distribution with *p* d.f.
3. $\sum_{i=1}^{n} (X_i - \mu)(X_i - \mu)'$ has Wishart distribution with *n* d.f.
4. $X_1 + X_2$ and $X_1 - X_2$ are independently distributed.

57. Suppose $X \sim N(0,1)$ and the conditional distribution of Y given $X = x$ is $N(\alpha x, 1)$, for $0 < \alpha < 1$. When we regress Y on X, the coefficient of determination R^2 is
1. α^2
2. α
3. $\frac{\alpha}{\sqrt{1+\alpha^2}}$
4. $\frac{\alpha^2}{1+\alpha^2}$

58. Let X and Y be integer-valued, bounded random variables. Then which of the following statement is FALSE ?
1. $E(X) = \sum_y E(X|Y=y) P(Y=y)$
2. $V(X) = \sum_y V(X|Y=y) P(Y=y)$
3. $P(X=x) = \sum_y P(X=x|Y=y) P(Y=y)$
4. $E(XY) = \sum_y y E(X|Y=y) P(Y=y)$

59. Suppose $X \sim$ Poisson $(\lambda), \lambda > 0$. Let the prior distribution of λ be $U(0,1)$. If

$X=0$ is observed, then the posterior probability of the set $0<\lambda\le\frac{1}{2}$ is

1. 0.5
2. 1
3. $\frac{e-\sqrt{e}}{e-1}$
4. $\frac{1}{e}$

60. Five persons A,B,C,D and E are seated at random on eight numbered chairs which are arranged in a circle. What is the probability that A and B are separated by at least 2 chairs?
1. 3/7
2. 1/2
3. 4/7
4. $\frac{1}{4}$

Part – C

Unit - I

61. Suppose A is a non-empty dense subset of $[0,1]$ then which of the following is not true :
1. A is compact
2. A is not compact
3. A is connected
4. A is not connected

62. The series $\sum_{n=2}^{\infty}\frac{\sin(nx)}{\log n}$ where $x\in\mathbb{R}$ is :
1. convergent for all x
2. divergent for all x
3. convergent for all $x\neq 0$
4. divergent for all $x\neq 0$

63. Let f be a differentiable function on (a,b) such that $f(n)=0$ and $\exists$ a real number $K>0$ such that $|f'(x)|\le K|f(x)|$ $\forall\ x\in[a,b]$ then which of the following is true :
1. $f(x)=0\ \forall\ x\in[a,b]$
2. $\exists\ x_0\in(a,b)$ such that $f(x_0)=0$
3. $\exists\ x_0\in(a,b)$ such that $f(x_0)\neq 0$
4. None of the above

64. Let $f:[0,2]\to\mathbb{R}$ be a function defined by

$$f(x)=\begin{cases}0 & \text{if} \quad x=\frac{n}{n+1} \text{ or } \frac{n+1}{n}, n\in\mathbb{N}\\ 1 & \text{if} \quad \text{otherwise}\end{cases}$$

Then which of the following is true :
1. f has countable discontinuities in $[0,2]$
2. f has uncountable discontinuities in $[0,2]$
3. f is Riemann integrable on $[0,2]$
4. f is continuous at $\sqrt{2}$

65. Let $f_n(x)=\frac{nx}{1+n^2x^2}$ and $S=\left\{x\in\mathbb{R}:|x|>\frac{1}{2}\right\}$ then which of the following is/are not true :
1. $\langle f_n\rangle$ is uniformly convergent on $\mathbb{R}$
2. $\langle f_n\rangle$ is uniformly convergent on S
3. Pointwise limit of $\langle f_n\rangle$ is continuous on both $\mathbb{R}$ and S.
4. Pointwise limit of $\langle f_n\rangle$ is continuous on S but not on $\mathbb{R}$

66. Suppose $f_n:[0,1]\to\mathbb{R}$ is defined by $f_n(x)=\frac{x^2}{x^2+(1-nx)^2}$ then which of the following is/are true :
1. The sequence $\langle f_n\rangle$ is uniformly bounded on $[0,1]$
2. $\lim_{n\to\infty}f_n(x)=0\ \forall\ x\in[0,1]$
3. $\langle f_n\rangle$ has a subsequence which is

uniformly convergent on $[0,1]$

4. $\langle f_n \rangle$ is uniformly convergent on $[0,1]$

67. For the sequence

$\langle a_n \rangle = \left\langle \frac{1}{2}, \frac{1}{3}, \frac{1}{2^2}, \frac{1}{3^2}, \frac{1}{2^3}, \frac{1}{3^3}, ... \right\rangle$.

Which of the following is true :

1. $\lim\inf \left(\frac{a_n + 1}{a_n} \right) = 0$
2. $\lim\sup \left(\frac{a_n + 1}{a_n} \right) = \infty$
3. $\lim\inf a_n^{\frac{1}{n}} = \frac{1}{\sqrt{3}}$
4. $\lim\sup a_n^{\frac{1}{n}} = \frac{1}{\sqrt{2}}$

68. Suppose $a_1 > 0$ and $a_{n+1} = \frac{3(1+a_n)}{5+a_n}, n \geq 1$

then which of the following is true :

1. $\langle a_n \rangle$ is decreasing if $a_1 > 1$
2. $\langle a_n \rangle$ is increasing if $a_1 < 1$
3. $\langle a_n \rangle$ is decreasing if $a_1 > 2$
4. $\langle a_n \rangle$ is increasing if $a_1 < \frac{1}{2}$

69. Let $f(x) = \frac{1}{e^x - 1}, x \neq 0$ and

$g(x) = \frac{1}{x}, x \neq 0$ then which of the following is/are not true :

1. $\lim_{x \to 0} f(x)$ does not exist.
2. $\lim_{x \to 0} g(x)$ does not exist.
3. $\lim_{x \to 0^+} (f(x) - g(x))$ does not exist.
4. $\lim_{x \to 0^-} (f(x) - g(x))$ does not exist

70. Rank of the matrix $\begin{bmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{bmatrix}$

is :

1. 3 if $a \neq 0, b \neq 0, c \neq 0$
2. 2 if $a = 0, b \neq 0, c \neq 0$
3. 2 if $a \neq 0, b = 0, c \neq 0$
4. 2 if $a = 0, b = 0, c \neq 0$

71. If A and B are non-singular matrices such that $AB - BA$ is singular then which of the following is/are true :

1. 0 must be an eigen value of $A^{-1}B^{-1}AB$.
2. 1 must be an eigen value of $A^{-1}B^{-1}AB$.
3. 1 can not be an eigen value of $A^{-1}B^{-1}AB$.
4. 0 can not be an eigen value of $A^{-1}B^{-1}AB$.

72. For the system of equation

$$x + 2y + z = 8$$
$$2x + y + 3z = 13$$
$$3x + 4y + \lambda y = \mu$$

Which of the following is/are true ?

1. No solution if $\lambda = \frac{11}{3}$ and $\mu \neq 22$
2. Unique solution if $\lambda \neq \frac{11}{3}$ and $\mu = 22$
3. Unique solution if $\lambda \neq \frac{11}{3}$ and $\mu \neq 22$

4. Infinite many solution if $\lambda = \frac{11}{3}$ and $\mu = 22$

73. Let A be a 5×5 complex matrix such that $A^4 = I$ then which of the following is/are not true :

1. All eigen values of A are distinct
2. A must have repeated eigen values
3. A is diagonalizable
4. One eigen value of A is of multiplicity 2.

74. Let V and W be finite dimensional vector spaces and let A be a linear transformation from V to W. Then which of the following is/are not true :

1. If $Av = 0$ only when $v = 0$, then $\dim V = \dim W$
2. If $\operatorname{Im} A = \{0\}$, then $A = 0$
3. If $V = W$ and $\operatorname{Im} A \subseteq \operatorname{Ker} A$, then $A = 0$
4. A is onto if and only if $\dim V \geq \dim W$

75. If A is a $n\times n$ real matrix such that $A^3 + A = 0$ then which of the following is/are true :

1. $\operatorname{tr}(A) = 0$ 2. $\operatorname{tr}(A) \neq 0$
3. $\det(A) = 0$ 4. $\det(A) \neq 0$

76. Which of the following is/are diagonalizable over $\mathbb{R}$?

1. $\begin{bmatrix} 3 & 1 & 2 \\ 1 & 4 & -3 \\ 2 & -3 & 7 \end{bmatrix}$ 2. $\begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$

3. $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$ 4. $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

77. Let A and B be square matrices of same size such that

$$M = \begin{pmatrix} A & B \\ B & A \end{pmatrix},\ N = \begin{pmatrix} A+B & 0 \\ 0 & A-B \end{pmatrix}$$

$$P = \begin{pmatrix} A & -B \\ B & A \end{pmatrix}, Q = \begin{pmatrix} A+iB & 0 \\ 0 & A-iB \end{pmatrix}$$

Then which of the following is/are true ?

1. M is similar to N
2. P is similar to Q
3. M is not similar to N
4. P is not similar to Q

78. Let V be a finite dimensional vector space and S be a subspace of V then which of the following is/are true :

1. $\dim(S) \leq \dim(V)$
2. $\dim(S) = \dim(V)$ iff $S = V$
3. Every basis for S is contained in some basis for V.
4. A basis for V need not contain a basis of S.

Unit – II

79. Let G be a group and $H = \{g^2 : g \in G\}$ then which of the following is/are true :

1. H is always a subgroup of G.
2. H may not be a subgroup of G.
3. If H is a subgroup of G then it must be normal.
4. H is a normal subgroup of G only if G is abelian.

80. Let S_4 be the symmetric group over four symbols and K is a normal subgroup of order 4 of S_4. If k_n denotes the number of

elements of order n in S_4 plus number of elements of order n in $\frac{S_4}{K}$ then

1. $k_2 = 10$
2. $k_3 = 10$
3. $k_2 = 12$
4. $k_3 = 9$

81. Which of the following polynomials are irreducible over $\mathbb{Q}$

1. $1 + x + x^2 + x^3 + x^4$
2. $1 - x + x^2 - x^3 + x^4$
3. $1 + x + x^2 + x^3 + x^4 + x^5 + x^6$
4. $1 - x + x^2 - x^3 + x^4 - x^5 + x^6$

82. Which of the following are units in ring $\mathbb{Z}\left(\sqrt{2}\right)$

1. $\left(7 + 5\sqrt{2}\right)^2$
2. $\left(7 + 5\sqrt{2}\right)^3$
3. $\left(7 + 5\sqrt{2}\right)^4$
4. $\left(7 + 5\sqrt{2}\right)^5$

83. Let $A = \mathbb{Q}\left(\sqrt{3}\right)$ and $B = \mathbb{Q}\left(\sqrt{-3}\right)$ then which of the following is/are true :

1. A and B are group-isomorphic
2. A and B are ring-isomorphic
3. A and B are field-isomorphic
4. A and B are vector space-isomorphic

84. Let $f(z) = u(x, y) + iv(x, y)$ be an analytic function in a domain D then which of the following real-valued functions are harmonic in D :

1. u
2. v
3. $u + v$
4. uv

85. Let C be a closed contour in the complex plane which is not passing through the origin and is not simple then which of the following values can be assumed by the integral $\int_C \frac{1}{z} dz$

1. 0
2. $2\pi i$
3. $2018\pi i$
4. $2019\pi i$

86. Which of the following is/are not true ?

1. A function can not have infinitely many isolated singularities in a bounded domain.
2. A function can not have infinitely many isolated singularities in a compact set of complex plane.
3. An entire function can not have uncountably many zeros.
4. A function can have infinitely many isolated singularities in the complex plane.

87. Which of the following is/are true :

1. There exists an entire function f such that $f(z) = z$ for all z such that $|z| = 1$ and $f(z) = 2z$ for all z such that $|z| = 2$
2. If f is analytic on $\{z : |z| < 2\}$ and $|f(z)|$ attains it maximum at the point $z = i$ then f must be constant.
3. If f is analytic on $\{z : |z| < 2\}$ and $|f(z)|$ attains its minimum at the point

$z = i$ then f must be constant.

4. If $S = \{z : 1 < |z| < 2\}$ and $f : \mathbb{C} \to S$ be an analytic function then f must be constant.

88. Consider the set $\mathbb{Z}$ of integers, with the topology τ in which a subset is closed if and only if it is empty, or $\mathbb{Z}$, or finite. Which of the following statements are true ?
    1. τ is the subspace topology induced from the usual topology on $\mathbb{R}$.
    2. $\mathbb{Z}$ is compact in the topology τ.
    3. $\mathbb{Z}$ is Hausdroff in the topology τ.
    4. Every infinite subset of $\mathbb{Z}$ is dense in the topology τ.

89. Consider the following subsets of the complex plane :

$$\Omega_1 = \left\{ C \in \mathbb{C} : \begin{bmatrix} 1 & C \\ \bar{C} & 1 \end{bmatrix} \text{is non-negative definite (or equivalently positive semi-definite)} \right\}$$

$$\Omega_2 = \left\{ C \in \mathbb{C} : \begin{bmatrix} 1 & C & C \\ \bar{C} & 1 & C \\ \bar{C} & \bar{C} & 1 \end{bmatrix} \text{is non-negative definite} \begin{pmatrix} \text{or equivalently positive} \\ \text{semi-definite} \end{pmatrix} \right\}$$

Let $\bar{D} = \{z \in \mathbb{C} \,|\, |z| \le 1\}$. Then

1. $\Omega_1 = \bar{D}, \Omega_2 = \bar{D}$
2. $\Omega_1 \ne \bar{D}, \Omega_2 = \bar{D}$
3. $\Omega_1 = \bar{D}, \Omega_2 \ne \bar{D}$
4. $\Omega_1 \ne \bar{D}, \Omega_2 \ne \bar{D}$

90. Let q_α and p_α $(\alpha = 1, 2, ..., n)$ be the generalized coordinates and the generalized momenta, respectively. If H denotes the Hamiltonian and q_a (for some $\alpha = \alpha_0$) is an ignorable coordinate, then which of the following equations are satisfied ?

1. $p_\alpha = -\dfrac{\partial H}{\partial q_\alpha}, \; q_\alpha = \dfrac{\partial H}{\partial p_\alpha}, \; \forall\, \alpha$
2. $p_\alpha = \dfrac{\partial H}{\partial q_\alpha}, \; q_\alpha = -\dfrac{\partial H}{\partial p_\alpha}, \; \forall\, \alpha$
3. $p_{\alpha_0} = 0, \; q_{\alpha_0} = \dfrac{\partial H}{\partial p_{\alpha_0}}$
4. $p_{\alpha_0} = \dfrac{\partial H}{\partial q_{\alpha_0}}, \; q_{\alpha_0} = 0$

Unit - III

91. The integral equation

$$\phi(x) = \lambda \int_0^1 \left(\sqrt{x}\,\xi - \sqrt{\xi}\,x\right)\phi(\xi)\,d\xi \text{ has}$$

1. real eigen function
2. no real eigen function
3. real eigen function
4. non-real eigen value

92. Given integral equation

$$\phi(x) = x + \int_0^x (\xi - x)\phi(\xi)\,d\xi$$

1. $K_2 = \dfrac{-1}{3!}(\xi - x)^3$
2. $K_3(x, \xi) = \dfrac{1}{5!}(\xi - x)^5$
3. The resolvent kernel is $R(x, \xi, 1) = \sin(\xi - x)$
4. The solution is $\phi(x) = \sin x$

93. Given $\sin x, \cos x, \sin 2x$, then

1. the wronskian of given equation is $3\sin 2x$
2. the wronskian of given equation is zero
3. the variables are linearly independent
4. the variables are linearly dependent

94. The extremals of the functional

$$I[y(x), z(x)] = \int_0^{\frac{\pi}{2}} \left(y'^2 + z'^2 + 2yz\right)dx,$$

satisfying

$$y(0) = 0, \; y\left(\frac{\pi}{2}\right) = 1, \; z(0) = 0, \; z\left(\frac{\pi}{2}\right) = -1$$

1. $y(x) = \sin x$ 2. $z(x) = -\sin x$

3. $y(x)=\sin^2 x$ 4. $z(x)=-\sin^2 x$

95. Find the shortest distance between the $x^2+y^2=1$ and the straight line $x+y=4$

1. $2\sqrt{2}+1$ 2. $2\sqrt{2}-1$
3. $2\sqrt{2}$ 4. None

96. $(D^2+4)y=\cos 2x$ solution is/are $(y_p=P.I.)$

1. $y_p=\frac{x}{4}\sin 2x$
2. $y_p=x\sin 2x$
3. $y(x)=c_1\cos 2x+c_2\sin 2x+x\sin 2x$
4. $y(x)=c_1\cos 2x+c_2\sin 2x+\frac{x}{4}\sin 2x$

97. Solution of ODE $(x+2y)(dx-dy)=dx+dy$ is/are

1. $3x-3y+a=2\log(3x+6y-1)$
2. $3x+3y+a=2\log(3x+6y-1)$
3. $3x-3y+a=-\log\left(\frac{1}{(3x+6y-1)^2}\right)$
4. None

98. $(x^2+2y^2)p-xyq=xz$ then solution is/are

1. $F\left(yz, y^2(x^2-y^2)\right)=0$
2. $F\left(xz, y^2(x^2+y^2)\right)=0$
3. $F\left(yz, y^2(x^2+y^2)\right)=0$
4. $yz=f\left(y^2(x^2-y^2)\right)$

99. $xp+yq=pq$ then

1. $z=\frac{(y+ax)^2}{2}+b$
2. $az=\frac{(y+ax)^2}{2}+b$
3. $z=\frac{(y+x)^2}{2}+b$
4. $z=\frac{(y+2x)^2}{4}+b$

100. $y''+\lambda y=0$; $y(0)=0$, $y'(1)=0$

1. $y_n(x)=b_n\sin(2n+1)\frac{\pi x}{2}$, where $n=0,1,2,......$
2. $\lambda\le 0$, No eigen function exist
3. $y_n(x)=a_n\sin(2n+1)\frac{\pi x}{2}$, where $n=1,2,3,....$
4. Every positive real number is a eigen value.

101. Consider a particle moving with coordinates $(x(t), y(t))$ on a smooth curve $\phi(x,y)=0$. If the particle moves from $(x(0), y(0))$ to $(x(\tau), y(\tau))$ for $\tau>0$ such that its kinetic energy is minimized, then

1. $\frac{\ddot{x}}{\phi_x}=\frac{\ddot{y}}{\phi_y}$
2. $\dot{x}^2(0)+\dot{y}^2(0)=\dot{x}^2(\tau)+\dot{y}^2(\tau)$
3. $\dot{x}\phi_x+\dot{y}\phi_y=0$
4. $\dot{x}^2(0)=\dot{x}^2(\tau)$

102. Which of the following approximation for estimating the derivative of a smooth function f at a point x is of order 2 (i.e. the error term is $O(h^2)$)

1. $f'(x) \approx \frac{f(x+h)-f(x)}{h}$
2. $f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$
3. $f'(x) \approx \frac{3f(x)-4f(x-h)+f(x-2h)}{2h}$
4. $f'(x) \approx \frac{-3f(x)+4f(x+h)-f(x+2h)}{2h}$

Unit - IV

103.Let $X_1, X_2, ..., X_n$, $n \geq 3$, be random sample from $N(\mu,1)$ population where μ is unknown. Define $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$.

Which of the following are necessarily true?

1. $\text{Cov}\left(X_1 - 3X_2 + 2X_3, \bar{X}_n\right) = 0$
2. Cramer-Rao lower bound for unbiased estimators of μ is $\frac{1}{n}$
3. $\text{Var}\left(\bar{X}_n\right) <$ $$\text{Var}\left(\frac{X_1 + 2X_2 + 3X_3 + ... + nX_n}{\frac{n(n+1)}{2}}\right)$$
4. $\bar{X}_n$ is a function of any sufficient statistic for μ.

104.Consider a Markov chain with state space $\{1,2,...,100\}$. Suppose states $2i$ and $2j$ communicate with each other and states $2i-1$ and $2j-1$ communicate with each other for every $i, j = 1,2,...,50$.

Further suppose that $p_{3,3}^{(2)} > 0, p_{4,4}^{(3)} > 0$ and $p_{2,5}^{(7)} > 0$. Then

1. The Markov chain is irreducible.
2. The Markov chain is aperiodic.
3. State 8 is recurrent.
4. State 9 is recurrent.

105.Let $X_1, X_2, ..., X_n$ be independent and identically distributed observations from the distribution with density

$f(x|\mu) = e^{-(x-\mu)}, x > \mu$ where $-\infty < \mu < \infty$ and μ is unknown. Let $T_1 = 2\sum_{i=1}^{n} X_i$ and $T_2 = 2X_{(1)}$ where $X_{(1)}$ is the smallest order statistic. To test $H_0 : \mu = 0$ versus $H_1 : \mu > 0$ at level α, where $0 < \alpha < 1$, consider the two tests A and B given below :

A : Reject H_0 if $T_1 > C_1$ where C_1 is such that $P(Y_1 > C_1) = \alpha$ with $Y_1 \sim \chi^2_{2n}$

B : Reject H_0 if $T_2 > C_2$ where C_2 is such that $P(Y_2 > C_2) = \alpha$ with $Y_2 \sim \chi^2_2$

Then which of the following statements are valid ?

1. Both A and B are level α tests.
2. A is the uniformly most powerful level α test.
3. B is the uniformly most powerful level α test.
4. B is more powerful than A at any $\mu > 0$

106.Let $X_1, X_2, X_3, ...$ be independent random variables with $E(X_k) = 0$ and $Var(X_k) = k$.

Let $S_n = \sum_{k=1}^{n} X_k$. Then, as $n \to \infty$,

1. $\frac{S_n}{n^{\frac{3}{2}}} \to 0$ in probability
2. $\frac{S_n}{n^{\frac{3}{2}}} \to 0$ in distribution
3. $\frac{S_n X_n}{n^{\frac{5}{2}}} \to 0$ in distribution
4. $\frac{S_n X_n}{n^{\frac{5}{2}}} \to 0$ in probability

107.Let $\phi(t)$ be a characteristic function of some random variable. Then, which of the following are also characteristic function ?

1. $f(t) = \left[\phi(t)\right]^2$ for all $t \in \mathbb{R}$.

2. $f(t)=|\phi(t)|^2$ for all $t\in\mathbb{R}$.
3. $f(t)=\phi(-t)$ for all $t\in\mathbb{R}$.
4. $f(t)=\phi(t+1)$ for all $t\in\mathbb{R}$.

108.Suppose X_1, X_2, X_3 and X_4 are independent and identically distributed random variables, having density function *f*. Then,

1. $P(X_4>\max(X_1,X_2)>X_3)=\frac{1}{6}$
2. $P(X_4>\max(X_1,X_2)>X_3)=\frac{1}{8}$
3. $P(X_4>X_3>\max(X_1,X_2))=\frac{1}{12}$
4. $P(X_4>X_3>\max(X_1,X_2))=\frac{1}{6}$

109. $N, A_1, A_2,....$ are independent real valued random variables such that $P(N=k)=(1-p)p^k,\ k=0,1,2,...$ where $0<p<1$, and $\{A_i : i=1,2,....\}$ is a sequence of independent and identically distributed bounded random variables. Let

$$X(w)=\begin{cases}0 & \text{if } N(w)=0\\ \sum_{j=1}^{k} A_j & \text{if } N(w)=k, k=1,2,...\end{cases}$$

Which of the following are necessarily correct?

1. X is a bounded random variable
2. Moment generating function m_X of X is $m_X(t)=\frac{(1-p)}{1-pm_A(t)}, t\in\mathbb{R}$, where m_A is the moment generating function of A_1.
3. Characteristic function φ_X of X is $\varphi_X(t)=\frac{(1-p)}{1-p\varphi_A(t)}, t\in\mathbb{R}$, where φ_A is the characteristic function of A_1.
4. X is symmetric about 0.

110.Suppose X is random variable such that $E(X)=0, E(X^2)=2$ and $E(X^4)=4$. Then

1. $E(X^3)=0$
2. $P(X\ge 0)=\frac{1}{2}$
3. $X \sim N(0,2)$
4. X is bounded with probability 1

111.Consider a 3-variate population with covariance matrix $\begin{pmatrix}\sigma^2 & \sigma^2\rho & 0\\ \sigma^2\rho & \sigma^2 & \sigma^2\rho\\ 0 & \sigma^2\rho & \sigma^2\end{pmatrix}$ where $\sigma^2>0, \rho>0$. Then which of the following statements are true ?

1. $\rho<\frac{1}{\sqrt{2}}$
2. The proportion of the total population variance explained by the first principal component is $\frac{1+2\rho}{3}$
3. The second principal component is uncorrelated with the first and the third principal component.
4. The proportion of the total population variance explained by the first two principal components is $\frac{\sqrt{2}}{3}(\rho+\sqrt{2})$

112.A and B are two methods to determine the levels of mercury in fish. In a study to compare A and B, amount of mercury was measured using both methods on $n=12$ fish. Let $(X_1,Y_1),....,(X_n,Y_n)$ be those measurements, with X_i's standing for method A and Y_i's for method B. It should be noted that the size of error in measurement can depend on the amount of mercury, so the observations $(X_1,Y_1),....,(X_n,Y_n)$ may not be identically distributed. To test

H_0: There is no difference between methods A and B

Versus

H_1: Method B typically gives a larger reading than method A,
which of the following test statistics are appropriate ?

1. Number of pairs (X_i, Y_j) with $(Y_j > X_i), 1 \le i \le n; 1 \le j \le n$
2. Sum of the ranks of the Y observations in the combined sample
3. Number of the pairs (X_i, Y_i) with $(Y_i > X_i), 1 \le i \le n$
4. $\bar{Y} - \bar{X}$

113.Let **Y** follow multivariate normal distribution $N_n(\mathbf{0}, \mathbf{I})$ and let A and B be $n \times n$ symmetric, idempotent matrices. Then which of the following statements are true ?

1. If $AB = 0$, then $\mathbf{Y}'A\mathbf{Y}$ and $\mathbf{Y}'B\mathbf{Y}$ are independently distributed.
2. If $Y'(A+B)\mathbf{Y}$ has chi-square distribution then $\mathbf{Y}'A\mathbf{Y}$ and $\mathbf{Y}'B\mathbf{Y}$ are independently distributed.
3. $\mathbf{Y}'(A-B)\mathbf{Y}$ has chi-square distribution.
4. $\mathbf{Y}'A\mathbf{Y}$ and $\mathbf{Y}'BY$ have chi-square distribution.

114.For a Markov chain with finite state space, the number of stationary distributions can be

1. 0
2. 1
3. 2
4. ∞

115.Let $X(t) =$ number of customers at time *t* in the system in an M/M/1 queueing model with arrival rate $\lambda > 0$ and service rate $\mu > 0$. Let

$\pi_k = \lim_{t \to \infty} P(X(t) = k), k = 0, 1, 2....$

whenever it exists.
Which of the following are true ?

1. $\{X(t)\}$ is a birth and death process with birth rates $\lambda_k = \lambda, k = 0, 1, 2, ...$ and death rates $\mu_k = \mu, k = 1, 2, ...$
2. $\{X(t)\}$ is a birth and death process with birth rates $\lambda_k = \frac{1}{\lambda}, k = 0, 1, 2, ...$ and death rates $\mu_k = \frac{1}{\mu}, k = 1, 2, ...$
3. Limiting distribution $\{\pi_k\}$ exists if and only if $\mu > \lambda$, and is the geometric distribution with parameter $\left(\frac{\lambda}{\mu}\right)$.
4. If an arriving customer finds exactly one customer, then his total waiting time in the system has an exponential distribution with parameter (2μ).

116.Consider the linear model

$y_1 = 2\theta + \beta + \epsilon_1$

$y_2 = \beta + 2\gamma + \epsilon_2$

$y_3 = \theta + \beta + \gamma + \epsilon_3$

Where θ, β, γ are unknown parameters and $\epsilon_1, \epsilon_2, \epsilon_3$ are uncorrelated random errors with mean 0 and constant variance. Then which of the following statements are true ?

1. θ, β and γ are estimable
2. $\theta - \gamma$ is estimable
3. $2\gamma - 2\theta$ is estimable
4. $\theta + \gamma$ is estimable

117.Suppose a sample of size *n* is drawn using simple random sampling without replacement from a finite population of N units where $N > n$ and denote the sample mean of the study variables corresponding to the selected units by $\bar{y}$. Now suppose we know one variate value y_1 corresponding to one unit and draw a simple random sample size *n* without replacement from the remaining $(N-1)$ units and denote the sample mean of the study variables corresponding to the selected units by $\bar{y}_0$. Define $t_1 = N\bar{y}$, $t_2 = (N-1)\bar{y}_0 + y_1, V_1 = \text{Var}(t_1)$ and $V_2 = \text{Var}(t_2)$. Which of the following are necessarily true ?

1. t_1 is unbiased for population total
2. t_2 is unbiased for population total

3. $V_1 = N^2 \frac{\sigma^2}{n} \frac{N-n}{N-1}$ where $\sigma^2 =$ population variance
4. $V_2 \le V_1$ for all n, N

118.Consider the linear model $E(Y) = X\beta, Cov(Y) = \sigma^2 I$ where X is a matrix of size $n \times p$ having rank $r \le p$. Then which of the following statements are necessarily true ?
1. The set of estimable linear functions form a vector space of dimension *r*.
2. If $E(c'Y) = 0$ for some nonzero vector *c*, then there is a function $l'\beta$ which is not estimable.
3. If all linear functions $l'\beta$ are estimable, then $r = p$
4. The set of functions $c'Y$ with $E(c'Y) = 0$ form a vector space of dimension *r*.

119.Consider a 2^3 factorial experiment with three factors A, B and C. Suppose eight treatments are assigned in two blocks of each of the four replicates in the following way.

Replicate 1		Replicate 2		Replicate 3		Replicate 4	
(1)	b	(1)	a	(1)	a	(1)	b
a	c	b	c	c	b	ab	a
bc	ac	ac	bc	ab	ac	bc	c
abc	ab	abc	ab	abc	bc	ac	abc

Which of the following are necessarily true ?
1. This is an example of complete confounding
2. AB is confounded in Replicate 1
3. AC is confounded in Replicate 2
4. ABC is confounded in Replicate 4

120.Let $X_1, X_2, ..., X_n$ be independent and identically distributed Bernoulli (θ), where $0 < \theta < 1$ and $n > 1$. Let the prior density of θ be proportional to $\frac{1}{\sqrt{\theta(1-\theta)}}, 0 < \theta < 1$. Define $S = \sum_{i=1}^{n} X_i$.
Then valid statements among the following are :
1. The posterior mean of θ does not exist;
2. The posterior mean of θ exists;
3. The posterior mean of θ exists and it is larger than the maximum likelihood estimator for all values of S.
4. The posterior mean of θ exists and it is larger than the maximum likelihood estimator for some values of S.

Answer Key

Part - A		Part - B				Part - C			
1.	2	21.	1	51.	2	61.	1,2,3,4	91.	2,4
2.	2	22.	3	52.	2	62.	1,3	92.	1,2,3,4
3.	4	23.	4	53.	2	63.	1,2	93.	1,3
4.	4	24.	2	54.	2	64.	1,3,4	94.	1,2
5.	2	25.	4	55.	3	65.	1,4	95.	2
6.	3	26.	4	56.	3	66.	1,2	96.	1,4
7.	3	27.	3	57.	4	67.	1,2,3,4	97.	1,3
8.	1	28.	1	58.	2	68.	1,2,3,4	98.	3
9.	2	29.	2	59.	3	69.	3,4	99.	2,3,4
10.	2	30.	4	60.	1	70.	1,2,3,4	100.	1,2
11.	2	31.	2			71.	2,4	101.	1,3
12.	4	32.	3			72.	1,2,3,4	102.	2,3,4
13.	4	33.	4			73.	1,4	103.	1,2,3,4
14.	3	34.	4			74.	1,3,4	104.	4
15.	3	35.	2			75.	1	105.	1,3,4
16.	2	36.	4			76.	1,3	106.	1,2,3,4
17.	3	37.	3			77.	1,2	107.	1,2,3
18.	4	38.	4			78.	1,2,3,4	108.	1,3
19.	2	39.	4			79.	2,3	109.	3
20.	3	40.	4			80.	2,3	110.	1,2,4
		41.	2			81.	1,2,3,4	111.	1,3,4
		42.	3			82.	1,2,3,4	112.	3
		43.	1			83.	1,4	113.	1,2,4
		44.	4			84.	1,2,3,4	114.	2,4
		45.	3			85.	1,2,3	115.	1,3
		46.	1			86.	1,2,3	116.	2,3
		47.	1			87.	2,4	117.	1,2,3,4
		48.	3			88.	2,4	118.	1,2,3
		49.	1			89.	3	119.	3,4
		50.	3			90.	1,3	120.	2,4